3.4 Use Linear Programming

Goal • Solve linear programming problems.

Your Notes

VOCABULARY

Constraints

Objective function

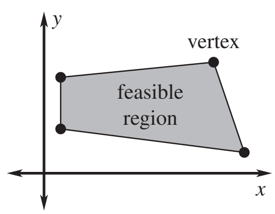
Linear programming

Feasible region

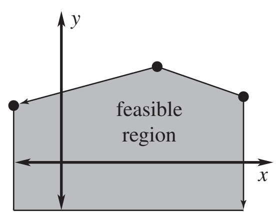
A feasible region is bounded if it is completely enclosed by line segments.

OPTIMAL SOLUTION OF A LINEAR PROGRAMMING PROBLEM

If the feasible region is , then the objective function has both a maximum and a minimum value on the region. Moreover, the maximum and minimum values each occur at a of the feasible region.



Bounded region



Unbounded region

3.4 Use Linear Programming

Goal • Solve linear programming problems.

Your Notes

VOCABULARY

Constraints Linear inequalities used to represent information about boundaries in a problem

Objective function A function to be maximized or minimized

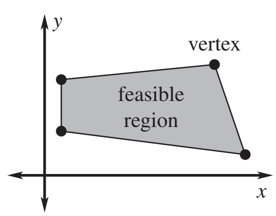
Linear programming The process of maximizing or minimizing a linear objective function subject to constraints that are linear inequalities

Feasible region All of the points in the intersection of the graphs of the constraints

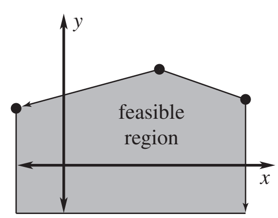
A feasible region is bounded if it is completely enclosed by line segments.

OPTIMAL SOLUTION OF A LINEAR PROGRAMMING PROBLEM

If the feasible region is **bounded**, then the objective function has both a maximum and a minimum value on the region. Moreover, the maximum and minimum values each occur at a **vertex** of the feasible region.



Bounded region

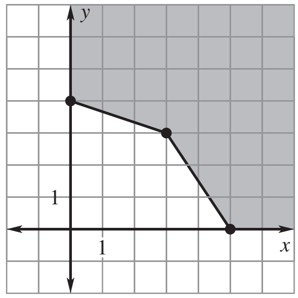


Unbounded region

Your Notes

Example 1 *Evaluate an objective function*

Find the minimum and the maximum value of the objective function $C = 3x + 4y$ for the given feasible region.



Solution

Step 1 **Identify** the vertices of the feasible region. The three vertices are (0, _), (_, _), and _____.

Step 2 **Evaluate** the objective function $C = 3x + 4y$ at each vertex of the feasible region.

At (0, _): C = 3(0) + 4(_) = __

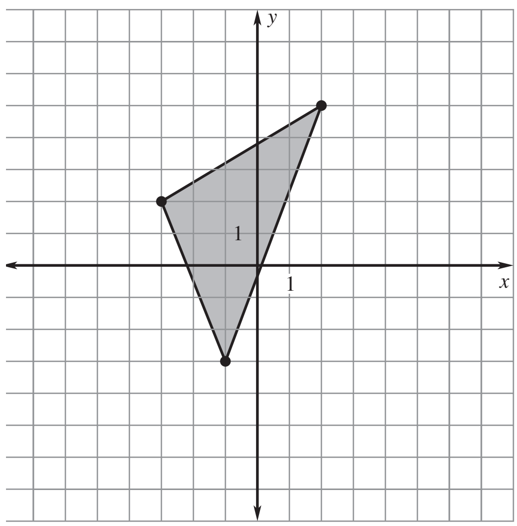
At (_, _): C = 3(3) + 4(_) = __

At (_, 0): C = ________ = __

The __________ value is __. Evaluating C at other points in the feasible region shows that as the points get _______ from the origin, the value of C ___________________________. So, the objective function has no __________ value.

Checkpoint **Complete the following exercise.**

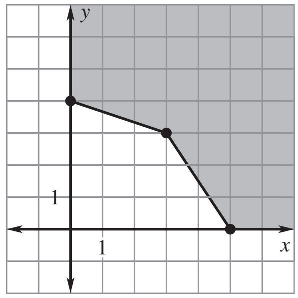
1. Find the minimum and the maximum value of the objective function $C = 2x + 7y$ for the given feasible region.



Your Notes

Example 1 *Evaluate an objective function*

Find the minimum and the maximum value of the objective function $C = 3x + 4y$ for the given feasible region.



Solution

Step 1 **Identify** the vertices of the feasible region. The three vertices are (0, 4), (3, 3), and (5, 0).

Step 2 **Evaluate** the objective function $C = 3x + 4y$ at each vertex of the feasible region.

At (0, 4): C = 3(0) + 4(4) = 16

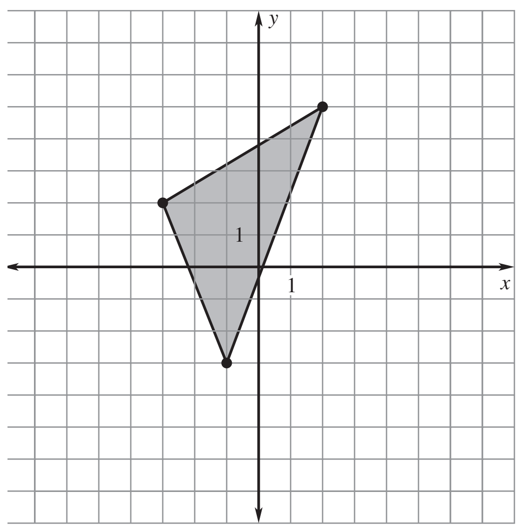
At (3, 3): C = 3(3) + 4(3) = 21

At (5, 0): C = 3(5) + 4(0) = 15

The minimum value is 15. Evaluating C at other points in the feasible region shows that as the points get farther from the origin, the value of C increases without bound. So, the objective function has no maximum value.

Checkpoint **Complete the following exercise.**

1. Find the minimum and the maximum value of the objective function $C = 2x + 7y$ for the given feasible region.



minimum: −23; maximum: 39

Your Notes

Example 2 ***Use linear programming to maximize profit***

Kim wants to knit and sell hats and scarves. A hat uses 6 ounces of yarn. A scarf uses 9 ounces. Kim has 72 ounces of yarn and wants to make at least 3 hats. The profit on a hat is $10 and the profit on a scarf is $20. How many hats and scarves should Kim make to maximize her profit?

Solution

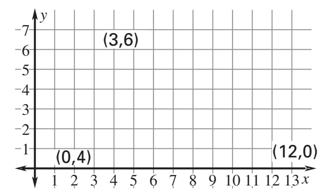
Step 1 **Write** an objective function for ___________ the profit P. Let x be the number of hats made and let y be the number of _______ made. The objective function is $P = 10x + 20y$.

Step 2 **Graph** the system of constraints.

______ Make at least ______.

______ Number of _______ cannot be negative.

____________ Can use up to __________ of yarn.



> You can find the coordinates of each vertex in the feasible region by solving systems of two linear equations. The solution of the system:
>
> $x = 3$
>
> $6x + 9y = 72$
>
> is the vertex (,)

Step 3 **Evaluate** the profit function $P = 10x + 20y$ at each vertex of the feasible region.

At (__, __): P = 10(__) + 20(__) = __

At (__, __): P = 10( ) + 20(__) = ___

At (__, __): P = _____________ = ___

Kim can maximize profits by making _ hats and _ scarves.

✔ *Checkpoint* Complete the following exercise.

2. Henry makes toy cars and trucks out of wood. It takes him 2 hours to make a car and 4 hours to make a truck. He makes a profit of $8 for each car sold and $18 for each truck sold. This week Henry wants to make at least 6 toy cars. If he has at most 60 hours available to make the cars and trucks, how many of each should he make in order to maximize his profit?

Your Notes

Example 2 *Use linear programming to maximize profit*

Kim wants to knit and sell hats and scarves. A hat uses 6 ounces of yarn. A scarf uses 9 ounces. Kim has 72 ounces of yarn and wants to make at least 3 hats. The profit on a hat is \$10 and the profit on a scarf is \$20. How many hats and scarves should Kim make to maximize her profit?

Solution

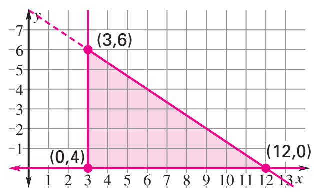
Step 1 **Write** an objective function for maximizing the profit P. Let x be the number of hats made and let y be the number of scarves made. The objective function is $P = 10x + 20y$.

Step 2 **Graph** the system of constraints.

$x \geq 3$ Make at least 3 hats.

$y \geq 0$ Number of scarves cannot be negative.

$6x + 9y \leq 72$ Can use up to 72 ounces of yarn.



You can find the coordinates of each vertex in the feasible region by solving systems of two linear equations. The solution of the system:

$x = 3$

$6x + 9y = 72$

is the vertex (3, 6)

Step 3 **Evaluate** the profit function $P = 10x + 20y$ at each vertex of the feasible region.

At (3, 0): P = 10(3) + 20(0) = 30

At (12, 0): P = 10(12) + 20(0) = 120

At (3, 6): P = 10(3) + 20(6) = 150

Kim can maximize profits by making 3 hats and 6 scarves.

Checkpoint **Complete the following exercise.**

2. Henry makes toy cars and trucks out of wood. It takes him 2 hours to make a car and 4 hours to make a truck. He makes a profit of \$8 for each car sold and \$18 for each truck sold. This week Henry wants to make at least 6 toy cars. If he has at most 60 hours available to make the cars and trucks, how many of each should he make in order to maximize his profit? 6 toy cars and 12 toy trucks

Your Notes

Example 3 ***Apply linear programming***

A company makes the same chair at two workshops. In one week, the company can use up to 105 hours of carpentry, 80 hours of machining, and 120 hours of finishing.

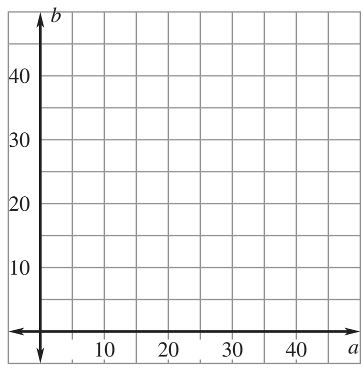
Resource	Hours per chair in Shop A	Hours per chair in Shop B
Carpentry	3	1
Machining	1	2
Finishing	3	2

Shop A earns $40 profit per chair and Shop B earns $30 profit per chair. How many chairs should the company make weekly in each shop to maximize its profit?

Solution

Step 1 **Write** an objective function for maximizing profit P. Let a and b represent chairs made in Shops A and B. The objective function is $P = 40a + 30b$.

Step 2 **Graph** the system of constraints.



______________ Carpentry up to ____ hr
______________ Machining up to ____ hr
_______________ Finishing up to ____ hr
______, _______ Production cannot be negative.

Step 3 **Evaluate** the profit at each vertex of the feasible region.

At (___, ____): P = 40(___) + 30(___) = ______
At (___, ___): P = 40(___) + 30(___) = ______
At (___, ___): P = 40(___) + 30(___) = ______
At (___, ___): P = _______________ = ______
At (___, ___): P = ______________ = ___

The company should make ___ chairs in Shop A and ___ chairs in Shop B.

Homework

✔ ***Checkpoint*** **Complete the following exercise.**

3. If shop A in Example 3 earned $50 profit per chair, how many chairs should the company make weekly in each shop?

Your Notes

Example 3 *Apply linear programming*

A company makes the same chair at two workshops. In one week, the company can use up to 105 hours of carpentry, 80 hours of machining, and 120 hours of finishing.

Resource	Hours per chair in Shop A	Hours per chair in Shop B
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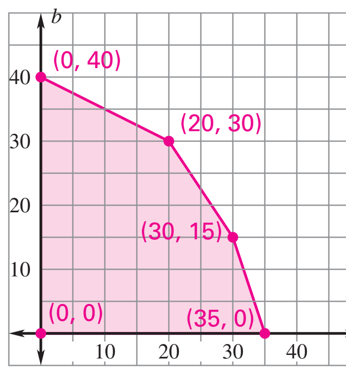

$3a + b \le 105$ Carpentry up to 105 hr

$a + 2b \le 80$ Machining up to 80 hr

$3a + 2b \le 120$ Finishing up to 120 hr

$a \ge 0$, $b \ge 0$ Production cannot be negative.

Step 3 **Evaluate** the profit at each vertex of the feasible region.

At (0, 40): $P = 40(0) + 30(40) = 1{,}200$

At (20, 30): $P = 40(20) + 30(30) = 1{,}700$

At (30, 15): $P = 40(30) + 30(15) = 1{,}650$

At (35, 0): $P = 40(35) + 30(0) = 1{,}400$

At (0, 0): $P = 40(0) + 30(0) = 0$

The company should make 20 chairs in Shop A and 30 chairs in Shop B.

Homework

Checkpoint Complete the following exercise.

3. If shop A in Example 3 earned \$50 profit per chair, how many chairs should the company make weekly in each shop?

   30 chairs in Shop A and 15 chairs in Shop B