Precalculus

Name __Key__
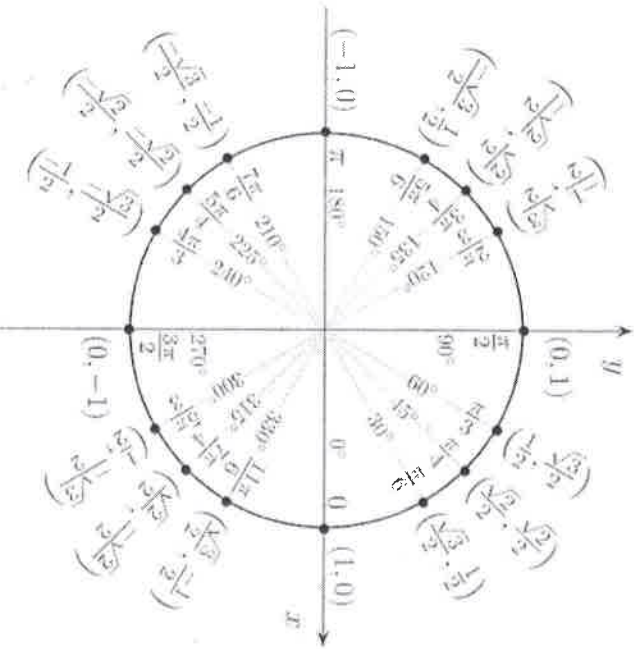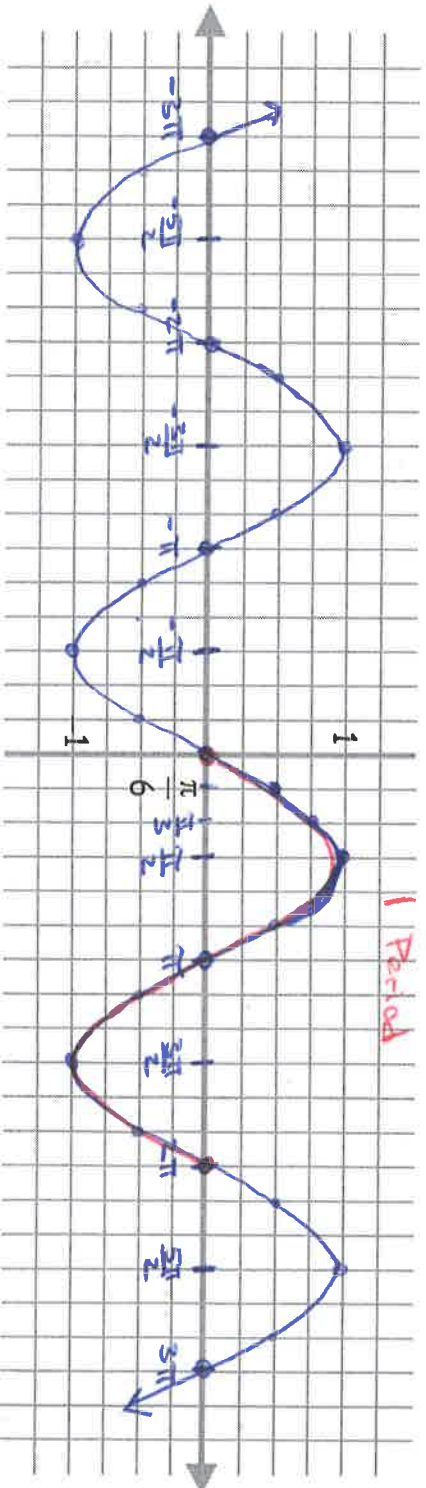
4.5 Notes: Graphs of Sine and Cosine Functions-Days 1 & 2

Graph the sine function. First complete the x-y chart by listing the corresponding radian measure for each degree measure. Strategically select angles (inputs) that have rational output values. Then, determine the y-value (output) for each x-value (input). Notice the pattern and continue graphing until the entire grid is filled. Use the unit circle to help you complete the table.

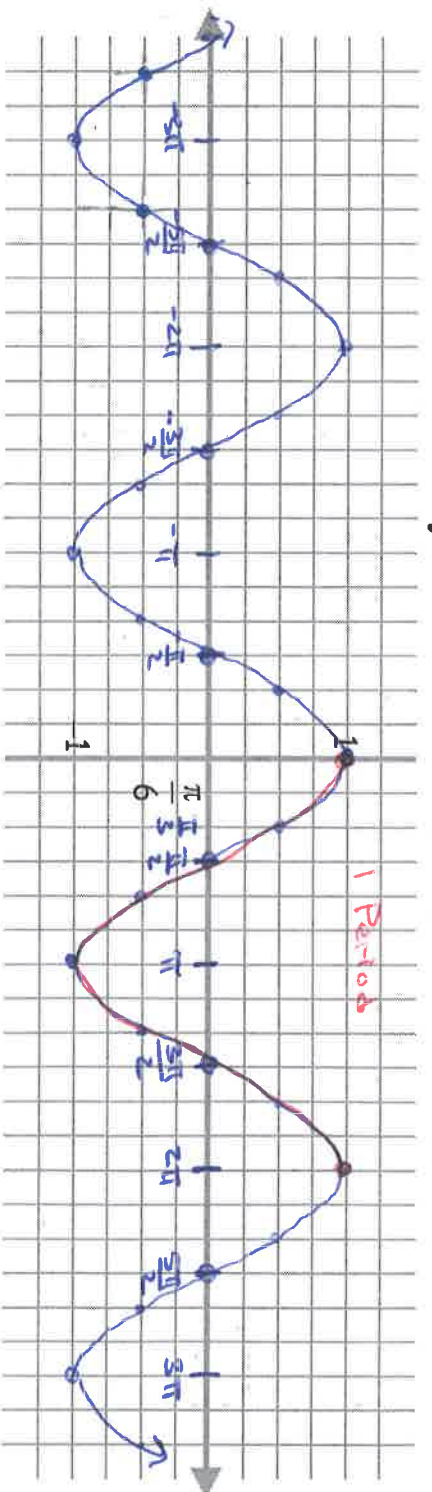| x radians	x degrees	y
-3π	$-540°$	0
$-\frac{5\pi}{2}$	$-450°$	-1
-2π	$-360°$	0
$-\frac{3\pi}{2}$	$-270°$	1
$-\pi$	$-180°$	0
$-\frac{\pi}{2}$	$-90°$	-1
0	0°	0
$\frac{\pi}{2}$	90°	1
π	180°	0
$\frac{3\pi}{2}$	270°	-1
2π	360°	0
$\frac{5\pi}{2}$	450°	1
3π	540°	0

$(\cos\theta, \sin\theta)$

$$y = \sin(x)$$

1 Period

- Domain: $(-\infty, \infty)$

- Range: $[-1, 1]$

- Period is the length of ONE CYCLE. Determine how many radians it takes before the sine curve starts to repeat?

  Period: 2π

- Amplitude is the *height* of the wave from the midline. For this parent function, the midline is the x-axis.
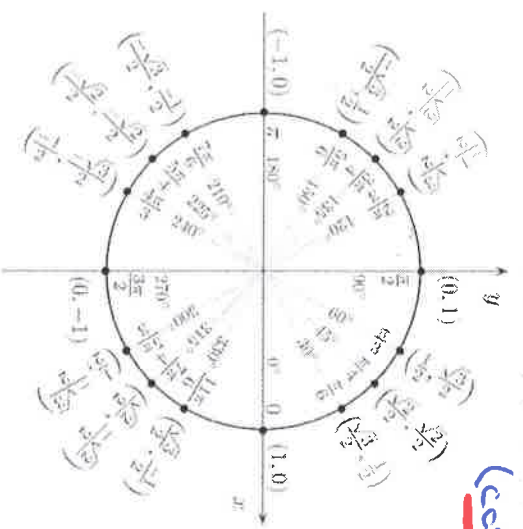
  Amplitude: __1__

  *always positive

Now, do the same thing for the <u>Cosine Function</u>.

$y = \cos(x)$

| x radians	x degrees	y
-3π	$-540°$	-1
$-\frac{5\pi}{2}$	$-450°$	0
-2π	$-360°$	-1
$-\frac{3\pi}{2}$	$-270°$	0
$-\pi$	$-180°$	-1
$-\frac{\pi}{2}$	$-90°$	0
0	$0°$	-1
$\frac{\pi}{2}$	$90°$	0
π	$180°$	1
$\frac{3\pi}{2}$	$270°$	0
2π	$360°$	1
$\frac{5\pi}{2}$	$450°$	0
3π	$540°$	-1

(cosθ, sinθ)

1 Period

➤ Domain: $(-\infty, \infty)$

➤ Range: $[-1, 1]$

➤ Period: 2π

➤ Amplitude: 1

Note: Functions that behave in such a repetitive (or cyclical) manner are called <u>Periodic</u>. The graphs of the sine and cosine functions are also known as <u>Sinusoidal</u> (wave) graphs.

Describe the transformation(s) that occurred to the parent function, $y = \sin x$.

1. $y = 2\sin x$
Vertically stretched by a factor of 2

2. $y = -\frac{1}{3}\sin x$
Reflected over x-axis
Vertically compressed by a factor of $\frac{1}{3}$

3. $y = \sin(x+1)$
Horizontal shift left 1 unit

4. $y = \sin x + 2$
Vertical shift up 2 units

5. $y = \sin(2x)$
Horizontal compression by a factor of $\frac{1}{2}$

6. $y = \sin\frac{x}{4}$
Horizontal stretch by a factor of 4

Use your knowledge of transformations to graph each of the following trigonometric functions. One period of the sine and cosine function are graphed below. Identify which one is which and use the "important values" as you translate each graph in questions #1-10.

increases   midline

Decreasing   midline
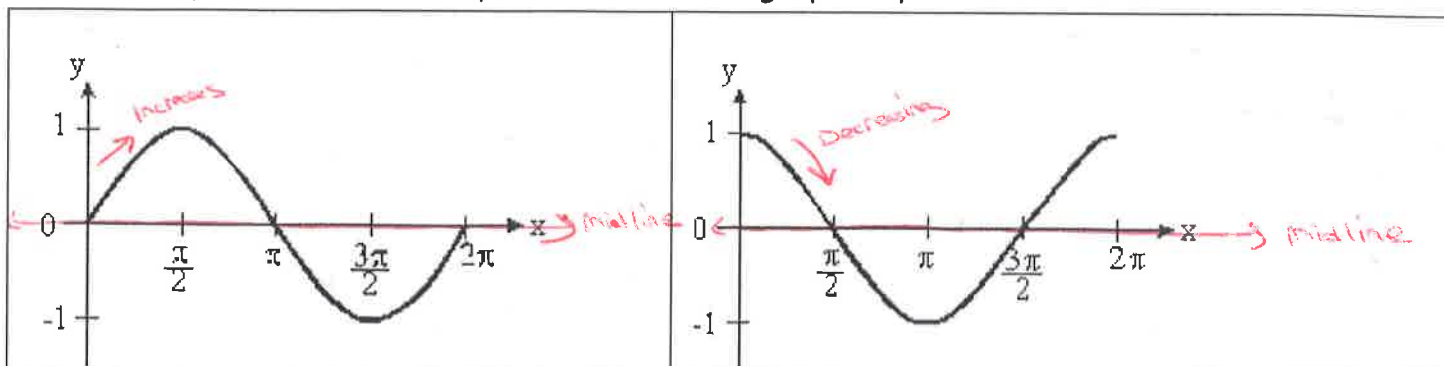
y = $\sin x$

y = $\cos x$

The "important values" are where the **peaks, valleys and intercepts** occur. The peaks and valleys are your **relative extrema**. For the sine and cosine function, these values happen every ____$\frac{\pi}{2}$____ radians. $\dfrac{\text{Period}}{4} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

The general equations for the sinusoidal functions are:

$$y = a\sin(bx - c) + d \quad \text{and} \quad y = a\cos(bx - c) + d$$

$|a|$ is the	$\dfrac{2\pi}{b}$ is the	$\dfrac{c}{b}$ is the	d is the
amplitude	Period	Phase shift	vertical shift
*Vert. stretch/comp.	*Horiz stretch/comp.	*Horiz shift w/ horiz stretch or comp.	*changes midline

Graph each of the following functions. Be sure to include all relative extrema and intercepts. Include at least ONE period for each function. List the amplitude and the period for each function. Use the indicated scale.
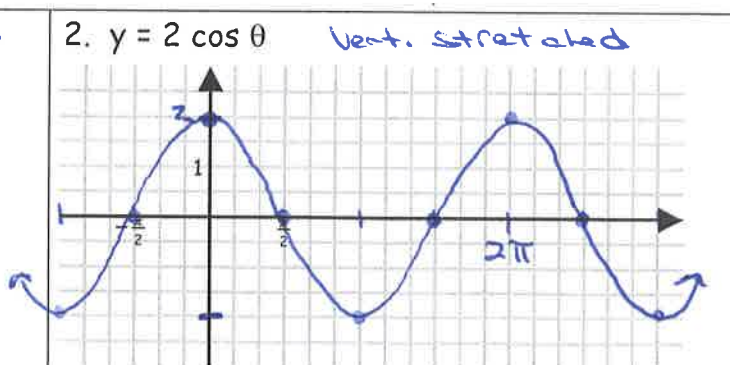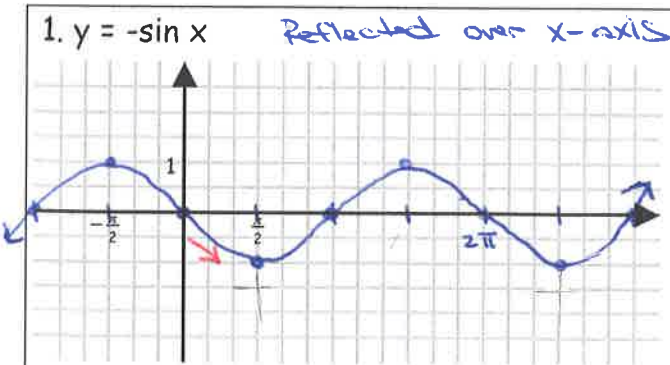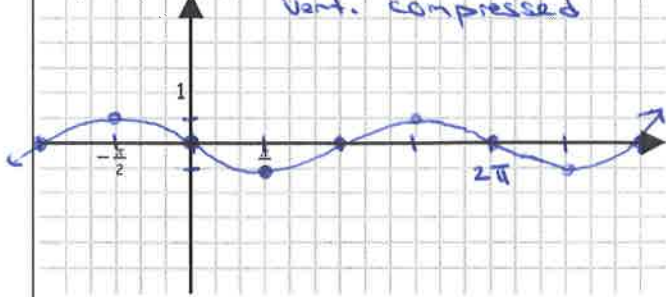
1. y = -sin x Reflected over x-axis	2. y = 2 cos θ Vert. stretched
Amplitude: 1 Period: 2π	Amplitude: 2 Period: 2π
Phase Shift: 0 "Important Values": every $\frac{\pi}{2}$	Phase Shift: 0 "Important Values": $\frac{\pi}{2}$

3. $y = -\frac{1}{2}\sin(\theta)$ — Reflected over x-axis, Vert. compressed
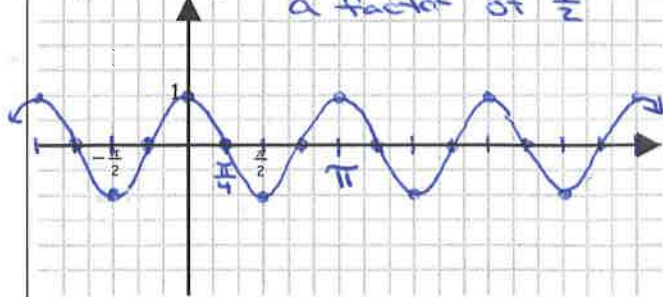
Amplitude: $\frac{1}{2}$ Period: 2π

Phase Shift: 0 "Important Values": every $\frac{\pi}{2}$
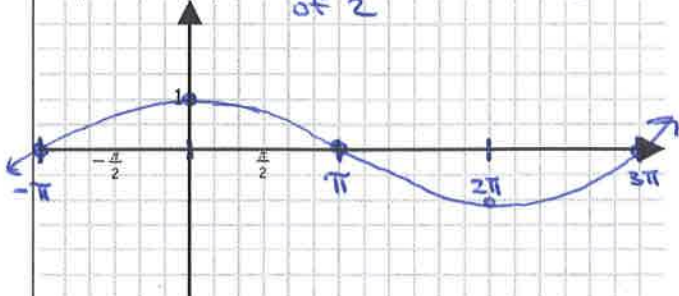
4. $y = \cos(2x)$ — Horiz. compression by a factor of $\frac{1}{2}$

Amplitude: 1 Period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Phase Shift: 0 "Important Values": every $\frac{\pi}{4}$
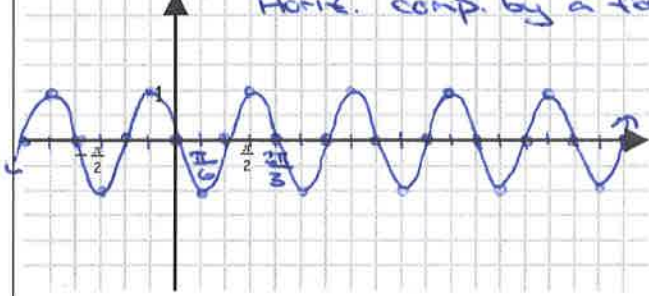
5. $y = \cos(\frac{1}{2}x)$ — Horiz. stretched by a factor of 2

Amplitude: 1 Period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$

Phase Shift: 0 "Important Values": every π
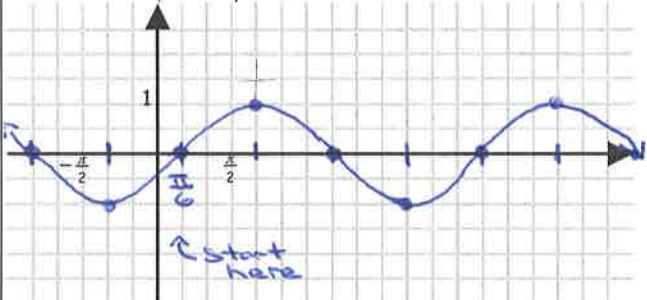
6. $y = -\sin(3\theta)$ — Reflected over x-axis, Horiz. comp. by a factor of $\frac{1}{3}$

Amplitude: 1 Period: $\frac{2\pi}{3}$

Phase Shift: 0 "Important Values": every $\frac{\pi}{6}$

$\frac{\frac{2\pi}{3}}{4}$ $\frac{2\pi}{3} \cdot \frac{1}{4}$ $\frac{\pi}{6}$

7. $y = \sin\left(x - \frac{\pi}{6}\right)$ — shift right $\frac{\pi}{6}$

↑ start here

Amplitude: 1 Period: 2π

Phase Shift: $\frac{c}{b} = \frac{\frac{\pi}{6}}{1} = \frac{\pi}{6}$ "Important Values": every $\frac{\pi}{2}$ or every $\frac{3\pi}{6}$ *common denom.
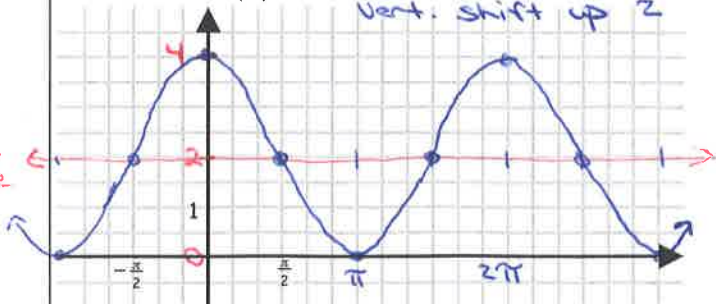
8. $y = \cos\left(\theta + \frac{\pi}{3}\right)$ — Shift left $\frac{\pi}{3}$
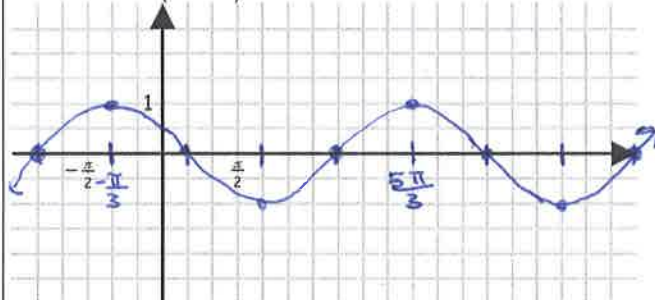
Amplitude: 1 Period: 2π

Phase Shift: $-\frac{\pi}{3}$ or $-\frac{2\pi}{6}$ "Important Values": every $\frac{\pi}{2}$ or $\frac{3\pi}{6}$

9. $y = 2\cos(\theta) + 2$ — Vert stretch by a factor of 2, Vert. shift up 2

mid-line →

Amplitude: 2 Period: 2π

Phase Shift: 0 Vertical Shift: 2

"Important Values": $\frac{\pi}{2}$ every *midline is up 2

10. $y = -\sin(x) - 1$ — Reflected over x-axis, Vert. shift down 1

→ Midline

Amplitude: 1 Period: 2π

Phase Shift: 0 Vertical Shift: -1

"Important Values": every $\frac{\pi}{2}$ *midline down 1