Pre-Calculus
Double Angle Formulas -HW Key Name ____________________

On notebook paper, work out each problem. For #1 – 6 circle your final answer.

1. Find the exact value of each using a sum or difference formula.

a. $\sin 195°$

$\sin(150°+45°)$

$= \sin 150° \cos 45° + \cos 150° \sin 45°$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$ or $\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

b. $\cos\frac{5\pi}{12}$

$\cos\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

c. $\tan 15°$

$\tan(45°-30°)$

$= \frac{\tan 45° - \tan 30°}{1 + (\tan 45°)(\tan 30°)}$

$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})}$

$= \frac{12-6\sqrt{3}}{6}$

$= \boxed{2-\sqrt{3}}$

2. Find $\sin 2x$ given $\sec x = 2$ when x is in quadrant IV.

$\sec x = 2$ means $\cos x = \frac{1}{2} = \frac{adj}{hyp}$ so $opp = \sqrt{3}$

$\sin 2x = 2\sin x \cos x = 2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{-\sqrt{3}}{2}}$

3. Find $\cos 2x$ given $\cot x = \frac{2}{3}$ when $\sin x > 0$. QI

$\cot x = \frac{2}{3} = \frac{adj}{opp}$ so $hyp = \sqrt{13}$

$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{2}{\sqrt{13}}\right)^2 - \left(\frac{3}{\sqrt{13}}\right)^2 = \frac{4}{13} - \frac{9}{13} = \boxed{\frac{-5}{13}}$

4. Find $\tan 2x$ given $\csc x = 4$ and $\tan x < 0$. Q2

$\csc x = 4$ means $\sin x = \frac{1}{4} = \frac{opp}{hyp}$ so $adj = \sqrt{15}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(-\frac{1}{\sqrt{15}}\right)}{1-\left(-\frac{1}{\sqrt{15}}\right)^2} = \frac{\frac{-2}{\sqrt{15}}}{1-\frac{1}{15}} = \frac{\frac{-2}{\sqrt{15}}}{\frac{14}{15}} = \frac{-2\sqrt{15}}{14} = \boxed{\frac{-\sqrt{15}}{7}}$

5. Find $\tan 2x$ given $\cos x = \frac{-4}{5}$, $\frac{\pi}{2} < x < \pi$. Q2

$\cos x = \frac{-4}{5} = \frac{adj}{hyp}$ so $opp = 3$ or $\sin x = \frac{3}{5}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2} = \frac{-\frac{3}{2}}{\frac{7}{16}} = \frac{-3}{2} \cdot \frac{16}{7} = \boxed{\frac{-24}{7}}$

6. Find $\cos 2x$ given $\sec x = \frac{-\sqrt{5}}{2}$, $\frac{\pi}{2} < x < \pi$. Q2

$\sec x = \frac{-\sqrt{5}}{2}$ means $\cos x = \frac{2}{-\sqrt{5}} = \frac{adj}{hyp}$ so $opp = 1$

$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{2}{-\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \boxed{\frac{3}{5}}$

7. Verify the identity.

$\cos(x-\pi) = -\cos x$

$\cos x \cos \pi + \sin x \sin \pi = RHS$

$\cos x(-1) + \sin x(0) = RHS$

$-\cos x = -\cos x$ ✓