NC Math 3 Honors – Unit 3A: Quadratics and Factoring Homework

3.1 Exponents and Operations with Polynomials

Simplify.

1. $\dfrac{2ab^5c}{a^3bc^2} = 2a^{-2}b^4c^{-1}$

   $\boxed{\dfrac{2b^4}{a^2c}}$

2. $\dfrac{xy^2}{2} \cdot \dfrac{6x}{y^2} = \dfrac{6x^2y^2}{2y^2} = \boxed{3x^2}$

3. $(s^2t)^3 \cdot st$

   $s^6t^3 \cdot st$

   $\boxed{s^7t^4}$

4. $(3x^4y^5)^{-3}$

   $\dfrac{1}{(3x^4y^5)^3} = \dfrac{1}{3^3x^{12}y^{15}} = \boxed{\dfrac{1}{27x^{12}y^{15}}}$

5. $\dfrac{(4x^2)^0}{2xy^5}$ $\boxed{\dfrac{1}{2xy^5}}$

6. $\dfrac{x^4x^{-2}}{x^{-5}} = \dfrac{x^2}{x^{-5}} = \boxed{x^7}$

Perform the indicated operation. Be sure to write your polynomial in standard form.

7. $(9y - 7x + 15a) + (-3y + 8x - 8a)$

   $\boxed{6y + x + 7a}$

8. $(9y - 7x + 15a) - (-3y + 8x - 8a)$

   $9y - 7x + 15a + 3y - 8x + 8a$

   $\boxed{12y - 15x + 23a}$

9. $(7a - 10b) - (3a + 4b)$

   $7a - 10b - 3a - 4b$

   $\boxed{4a - 14b}$

10. $(2x + 3)(5x + 8)$

    $10x^2 + 16x + 15x + 24$

    $\boxed{10x^2 + 31x + 24}$

11. $(y + 4)(y - 3)$

    $y^2 - 3y + 4y - 12$

    $\boxed{y^2 + y - 12}$

12. $(3y + 5)(2y - 6)$

    $6y^2 - 18y + 10y - 30$

    $\boxed{6y^2 - 8y - 30}$

13. $(2x + 3)(2x + 3)$

    $4x^2 + 6x + 6x + 9$

    $\boxed{4x^2 + 12x + 9}$

14. $(x - 5)(x + 5)$

    $x^2 + 5x - 5x - 25$

    $\boxed{x^2 - 25}$

3.2 Factoring Using GCF, Grouping, and Difference of Squares

Factor the GCF (greatest common factor) from each polynomial.

1. $45x^2 - 25$

   $\boxed{5(9x^2 - 5)}$

2. $27x^2y^5 - 72x^3y^2$

   $\boxed{9x^2y^2(3y^3 - 8x)}$

3. $-30b^9 + 5ab - 15a^2$

   $\boxed{-5(6b^9 - 5ab + 3a^2)}$

4. $30qpr - 5qp + 50q$

   $\boxed{5q(6pr - p + 10)}$

5. $-10x^4 + 20y^2 + 12x$

   $\boxed{-2(5x^4 - 10y^2 - 6x)}$

6. $x - 5$

   $\boxed{x - 5}$

Factor each polynomial by grouping.

7. $12a^3 - 9a^2 + 4a - 3$

$3a^2(4a-3) + 1(4a-3)$

$\boxed{(4a-3)(3a^2+1)}$

8. $35xy - 5x - 56y + 8$

$5x(7y-1) - 8(7y-1)$

$\boxed{(7y-1)(5x-8)}$

9. $3n^3 - 4n^2 + 9n - 12$

$n^2(3n-4) + 3(n-4)$

$\boxed{(n^2+3)(3n-4)}$

10. $9xz - 15z - 21xu + 35u$

$3z(3x-5) - 7u(3x-5)$

$(3x-5)(3z - 7u)$

$\boxed{(3x-5)(3z-7u)}$

11. $5n^3 - 10n^2 + 3n - 6$

$5n^2(n-2) + 3(n-2)$

$\boxed{(n-2)(5n^2+3)}$

Factor each difference of squares.

12. $x^2 - 100$

$\boxed{(x+10)(x-10)}$

13. $4a^2 - 16$

$4(a^2-4)$

$\boxed{4(a+2)(a-2)}$

14. $-3b^4 + 27$

$-3(b^4-9) = -3\left((b^2)^2 - 9\right)$

$\boxed{-3(b^2+3)(b^2-3)}$

15. $h^4 - 81$

$(h^2+9)(h^2-9)$

$\boxed{(h^2+9)(h+3)(h-3)}$

3.3 Factoring Sum and Difference of Cubes, Trinomials

Factor each sum or difference of cubes.

1. $27x^3 - 27$

$27(x^3-1)$

$\boxed{27(x-1)(x^2+x+1)}$

2. $64x^3 + 125 = 4^3x^3 + 5^3$

$\boxed{(4x+5)(16x^2-20x+25)}$

3. $27x^3 - 8y^3 = 3^3x^3 - 2^3y^3$

$\boxed{(3x-2y)(9x^2+6xy+4y^2)}$

4. $-x^3 - 343$

$-1(x^3+343) = -1(x^3 - 7^3)$

$\boxed{-1(x-7)(x^2+7x+49)}$

Factor each trinomial.

5. $6c^2 + 11c + 4$

$8 \diagup\!\!\!\!\diagdown 3 \quad \boxed{(2c+1)(3c+4)}$

$6c^2 + 8c + 3c + 4$

$2c(3c+4) + 1(3c+4)$

$(3c+4)(2c+1)$

6. $4x^2 + 2x - 12$

$2(2x^2+x-6)$

$\boxed{2(2x-3)(x+2)}$

7. $2m^2 + m - 28$

$\boxed{(2m-7)(m+4)}$

8. $x^2 - 18x + 81$

$\boxed{(x-9)(x-9)}$

or

$\boxed{(x-9)^2}$

9. $x^2 - 8x + 15$
$$(x-5)(x-3)$$

11. $9x^2 + 48x + 64$
$(3x+8)(3x+8)$
$$(3x+8)^2$$

10. $x^2 - 3x - 40$
$$(x-8)(x+5)$$

12. $x^2 - 2xy + y^2$
$(x-y)(x-y)$
$$(x-y)^2$$

3.4 Factoring Polynomials

Completely factor each polynomial.

1. $36x^2 - 144y^2$
$36(x^2-4y^2)$
$$36(x-2y)(x+2y)$$

10. $x^2 + 10x + 25$
$(x+5)(x+5)$
or
$$(x+5)^2$$

19. $x^4 - y^4$
$(x^2+y^2)(x^2-y^2)$
$$(x^2+y^2)(x+y)(x-y)$$

2. $8x^2 - 8x$
$$8x(x-1)$$

11. $7x - 49y$
$$7(x-7y)$$

20. $x^3 - x^2 - 4x + 4$
$x^2(x-1)-4(x-1)$
$(x-1)(x^2-4)$
$$(x-1)(x+2)(x-2)$$

3. $m^2 - 225$
$$(m+15)(m-15)$$

12. $2x^2 + x - 21$
$$(2x+7)(x-3)$$

21. $a^2 + 6a + 9 - y^2$
$a(a+6)?$
$$not\ factorable$$

4. $16y^2 + 2y - 3$ $8 \overset{48}{\underset{2}{\times}} 6$
$$(8y-3)(2y+1)$$
$16y^2+8y-6y-3$
$8(2y+1)-3(2y+1)$
$(2y+1)(8y-3)$

13. $8y^3 + 8z^3$
$8(y^3+z^3)$
$$8(y+z)(y^2-yz+z^2)$$

22. $b^3 - 3b^2 + 4b - 12$
$b^2(b-3)+4(b-3)$
$$(b^2+4)(b-3)$$

5. $8y^2 - 4$
$$4(2y^2-1)$$

14. $8p^2 - 64$
$$8(p^2-8)$$

23. $2b^2x - 50x$
$2x(b^2-25)$
$$2x(b+5)(b-5)$$

6. $m^3 + 8$

$m^3 + 2^3$

$\boxed{(m+3)(m^2 - 2m + 4)}$

15. $12y^2 - 14y - 20$

$2(6y^2 - 7y - 10)$

$\boxed{2(6y+5)(y-2)}$

24. $-2z^2 + 11z + 21$

$-(2z^2 - 11z - 21)$

$\boxed{-(2z+3)(z-7)}$

7. $3x^2 - 28x + 32$

$\boxed{\text{not factorable}}$

16. $4x^2 - 49$ $(2x)^2 - 7^2$

$\boxed{(2x+7)(2x-7)}$
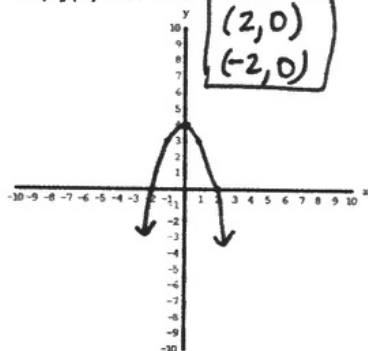
25. $21 - 7y + 3x - xy$

$7(3-y) + x(3-y)$

$\boxed{(3-y)(7+x)}$

3.5 Solving Quadratics by Factoring and Graphing

For 1 – 2, solve by graphing.

1.) $f(x) = -x^2 + 4$

$\boxed{\begin{array}{c}(2,0)\\(-2,0)\end{array}}$

2.) $h(x) = -x^2 - 2x + 8$

$\boxed{\begin{array}{c}(-4,0)\\(2,0)\end{array}}$
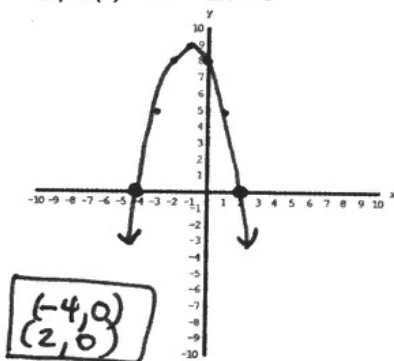
$x = -\dfrac{b}{2a} = \dfrac{-(-2)}{2(-1)} = -1$

$h(-1) = -(-1)^2 - 2(-1) + 8$

$h(-1) = -1 + 2 + 8 = 9$

vertex $(-1, 9)$

X	Y
-3	5
-2	8
-1	9
0	8
1	5
2	0

For 3 - 4, a quadratic function and its graph are shown. Identify the solutions, or roots, of the related quadratic equation.

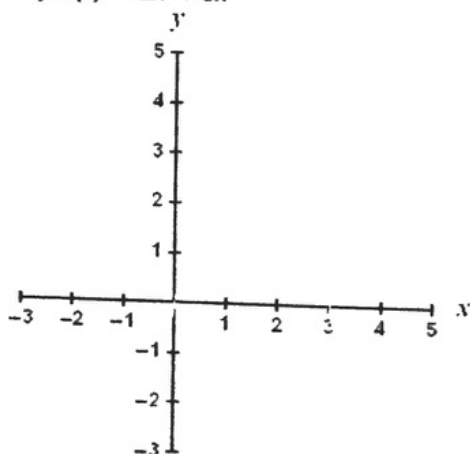**3.)** $g(x) = 3x^2 - 3x - 6$
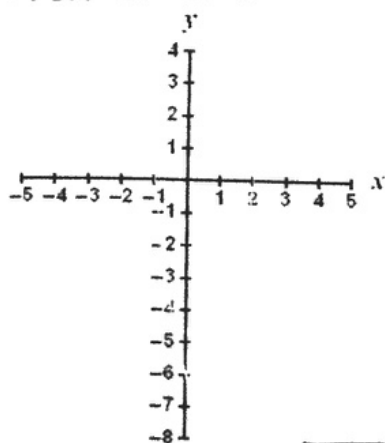
Solve: $0 = 3x^2 - 3x - 6$ $\boxed{x = \underline{2}\ \text{or}\ \underline{-1}}$

$0 = 3(x^2 - x - 2)$

$0 = 3(x-2)(x+1)$

$x - 2 = 0 \quad x + 1 = 0$

4.) $h(x) = -2x^2 + 6x$

Solve: $0 = -2x^2 + 6x$ $\boxed{x = \underline{0}\ \text{or}\ \underline{3}}$

$-2x(x-3)$

$x = 0 \quad x - 3 = 0$

Solve by square roots or factoring.

5. $z^2 - 5z + 4 = 0$

$(z-4)(z-1) = 0$

$z-4=0 \quad z-1=0$

$\boxed{z = 1, 4}$

6. $c^2 + 6c + 5 = 0$

$(c+5)(c+1) = 0$

$c+5=0 \quad c+1=0$

$\boxed{c = -5, -1}$

7. $x^2 - 49 = 0$ OR $\sqrt{x^2} = \sqrt{49}$

$(x+7)(x-7) = 0$ $\boxed{x = \pm 7}$

$x+7=0 \quad x-7=0$

$\boxed{x = 7, -7}$

8. $3x^2 - 12 = 0$ OR $3x^2 = 12$

$3(x^2-4) = 0$ $x^2 = 4$

$3(x+2)(x-2) = 0$ $\boxed{x = \pm 2}$

$\boxed{x = -2, 2}$

9. $d^2 - 2d = 0$

$d(d-2) = 0$

$\boxed{d=0} \quad d-2=0$

        $\boxed{d=2}$

10. $2x^2 - 5x + 2 = 0$

$(2x-1)(x-2) = 0$

$2x-1=0 \quad x-2=0$

$\boxed{x = \frac{1}{2}, 2}$

11. $5x^2 + 11x + 2 = 0$

$(5x+1)(x+2) = 0$

$5x+1=0 \quad x+2=0$

$\boxed{x = -\frac{1}{5}, -2}$

   $5x^2 + 10x + x + 2 = 0$

   $5x(x+2) + 1(x+2)$

   $(5x+1)(x+2)$

12. $y^2 = 8y + 20$

$y^2 - 8y - 20 = 0$

$(y-10)(y+2) = 0$

$\boxed{y = 10, -2}$

13. $x^2 + 3x - 4 = 50$

$x^2 + 3x - 54 = 0$

$(x+9)(x-6) = 0$

$\boxed{x = -9, 6}$

14. $2x^2 + 7 = 5 - 5x$

$2x^2 + 5x + 2 = 0$

$(2x+1)(x+2) = 0$

$\boxed{x = -\frac{1}{2}, -2}$

15. $x(x-2) = 35$

$x^2 - 2x = 35$

$x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$\boxed{x = 7, -5}$

16. $(x+2)^2 = 64$

$x^2 + 4x + 4 = 64$

$x^2 + 4x - 60 = 0$

$(x+10)(x-6) = 0$

$\boxed{x = -10, 6}$

OR $\sqrt{(x+2)^2} = \sqrt{64}$

$x+2 = \pm 8$

$x = -2 \pm 8$

$x = -2+8 = \boxed{6}$

$x = -2-8 = \boxed{-10}$

3.6 Quadratic Formula, Radicals and Complex Numbers

Simplify each radical.

1. $\sqrt{20} = \sqrt{4} \cdot \sqrt{5}$

    $\boxed{2\sqrt{5}}$

2. $\sqrt{72}$

$\sqrt{36} \cdot \sqrt{2}$

$\boxed{6\sqrt{2}}$

3. $\sqrt{108a^3b^2}$

$\sqrt{108} = \sqrt{36} \cdot \sqrt{3}$

$\boxed{6ab\sqrt{3a}}$

4. $\sqrt{567x^5yz^3}$

$\boxed{24x^2z\sqrt{xyz}}$

5. $\sqrt{-8}$ $i\sqrt{8}$

$i\sqrt{4} \cdot \sqrt{2}$

$\boxed{2i\sqrt{2}}$

6. $\sqrt{-45}$ $i\sqrt{45}$

$i\sqrt{9} \cdot \sqrt{5}$

$\boxed{3i\sqrt{5}}$

7. $-\sqrt{-200}$

$-i\sqrt{200}$

$-i\sqrt{100} \cdot \sqrt{2}$

$\boxed{-10i\sqrt{2}}$

8. $\sqrt{-10}$

$\boxed{i\sqrt{10}}$

Solve each quadratic equation using the quadratic formula. Be sure to simplify your answer.

9. $x^2 - x - 20 = 0$

$X = \dfrac{1 \pm \sqrt{(-1)^2 - 4(1)(-20)}}{2(1)} = \dfrac{1 \pm \sqrt{81}}{2} = \dfrac{1 \pm 9}{2}$

$\dfrac{10}{2} = \boxed{5}$ or $\dfrac{-8}{2} = \boxed{-4}$

10. $x^2 - 1 = 0$

$X = \dfrac{0 \pm \sqrt{0^2 - 4(1)(-1)}}{2(1)} = \dfrac{\pm\sqrt{4}}{2} = \pm\dfrac{2}{2} =$

$\boxed{\pm 1}$

11. $a^2 - 2a + 5 = 0$

$X = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)} = \dfrac{2 \pm \sqrt{-16}}{2}$

$X = \dfrac{2 \pm 4i}{2} = \boxed{1 \pm 2i}$

12. $2a^2 - a + 1 = 0$

$X = \dfrac{1 \pm \sqrt{(-1)^2 - 4(2)(1)}}{2(2)} = \dfrac{1 \pm \sqrt{-7}}{4}$

$\boxed{X = \dfrac{1 \pm 7i}{4}}$

13. $t^2 + 1 = 0$

$X = \dfrac{0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \dfrac{\pm\sqrt{-4}}{2} = \dfrac{\pm 2i}{2}$

$\boxed{X = \pm i}$

14. $x^2 - 7x + 9 = 0$

$X = \dfrac{7 \pm \sqrt{(-7)^2 - 4(1)(9)}}{2(1)}$

$X = \dfrac{7 \pm \sqrt{49 - 36}}{2}$

$\boxed{X = \dfrac{7 \pm \sqrt{13}}{2}}$

15. $2x^2 + 3x - 1 = 0$

$X = \dfrac{-3 \pm \sqrt{(3)^2 - 4(2)(-1)}}{2(2)} = \dfrac{-3 \pm \sqrt{17}}{4}$

$\boxed{X = \dfrac{-3 \pm \sqrt{17}}{4}}$

16. $c^2 - 2x + 4 = 0$

$X = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = \dfrac{2 \pm \sqrt{-12}}{2}$ $\quad \boxed{\dfrac{\sqrt{12}}{\sqrt{4}\cdot\sqrt{3}} \\ 2\sqrt{3}}$

$X = \dfrac{2 \pm 2i\sqrt{3}}{2} = \boxed{1 \pm i\sqrt{3}}$

17. $5n^2 + 3n + 1 = 0$

$X = \dfrac{-3 \pm \sqrt{(3)^2 - 4(5)(1)}}{2(5)} = \dfrac{-3 \pm \sqrt{-11}}{10}$

$\boxed{X = \dfrac{-3 \pm i\sqrt{11}}{10}}$

18. $k^2 - 5k - 24 = 0$

$k = \dfrac{5 \pm \sqrt{(-5)^2 - 4(1)(-24)}}{2(1)} = \dfrac{5 \pm \sqrt{121}}{2} = \dfrac{5 \pm 11}{2}$

$k = \dfrac{16}{2}$ or $\dfrac{-6}{2}$; $\boxed{8, -3}$

19. $4y^2 - 8y + 3 = 0$

$Y = \dfrac{8 \pm \sqrt{(-8)^2 - 4(4)(3)}}{2(4)} = \dfrac{8 \pm \sqrt{16}}{8}$

$Y = \dfrac{8 \pm 4}{8} \quad \boxed{Y = \dfrac{3}{2}, \dfrac{1}{2}}$

20. $6x^2 + 3x + 2 = 0$

$X = \dfrac{-3 \pm \sqrt{3^2 - 4(6)(2)}}{2(6)}$

$X = \dfrac{-3 \pm \sqrt{-39}}{12}$

$\boxed{X = \dfrac{-3 \pm i\sqrt{39}}{12}}$