Key

Name:_____

Pre-Calculus
Analytical Trig REVIEW

Objective: Use trigonometric identities to verify identities and solve equations.

Verify the identity.

1. $\dfrac{\tan x}{\sec x} = \sin x$

$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \sin x =$ ✓

2. $\sin x + \cot x \cos x = \csc x$

$\dfrac{\sin x}{1} + \dfrac{\cos x}{\sin x} \cdot \cos x = \dfrac{1}{\sin x}$

$\dfrac{\sin^2 x + \cos^2 x}{\sin x} = \dfrac{1}{\sin x} =$ ✓

3. $\sin(-x)\cot(-x) = \cos x$

$(-\sin x)\left(-\dfrac{\cos x}{\sin x}\right) = \cos x$

$\cos x =$ ✓

4. $\dfrac{1}{1+\cos(-x)} + \dfrac{1}{1-\cos x} = 2\csc^2 x$

$\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x} = 2\csc^2 x$

$\dfrac{1-\cos x + 1+\cos x}{1-\cos^2 x} = \dfrac{+2}{\sin^2 x} = 2\csc^2 x =$ ✓

$\dfrac{2}{1} \cdot \dfrac{1}{\sin^2 x} =$

5. $\tan x \cos x + \csc x \sin^2 x = 2\sin x$

$\dfrac{\sin x}{\cos x} \cdot \cos x + \dfrac{1}{\sin x} \cdot \sin^2 x = 2\sin x$

$\sin x + \sin x = 2\sin x =$ ✓

Objective: Use angle formulas to find exact values.

6. $\cos\left(\dfrac{\pi}{12}\right) = \cos\left(\dfrac{\pi}{6} - \dfrac{\pi}{4}\right)$ $= \cos\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

7. $\sin 23^\circ \cos 67^\circ + \cos 23^\circ \sin 67^\circ$ sin, + so it's $\sin(90^\circ) = \boxed{1}$

8. $\sin\left(\dfrac{\pi}{3} - \dfrac{\pi}{6}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{6} - \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \boxed{\frac{1}{2}}$

9. $\cos\left(\dfrac{5\pi}{4} + \dfrac{\pi}{2}\right) = \cos\frac{5\pi}{4}\cos\frac{\pi}{2} - \sin\frac{5\pi}{4}\sin\frac{\pi}{2}$

$= -\frac{\sqrt{2}}{2}\cdot 0 - \left(-\frac{\sqrt{2}}{2}\cdot 1\right) = \boxed{\frac{\sqrt{2}}{2}}$

10. $\dfrac{\tan 83^\circ - \tan 23^\circ}{1 + \tan 83^\circ \tan 23^\circ}$

$= \tan 60^\circ = \boxed{\sqrt{3}}$

11. Given $\sin\alpha = \dfrac{4}{5}$ and α is in quadrant I, find $\cos(2\alpha)$.

3-4-5

$= \cos^2 x - \sin^2 x$

$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9-16}{25} = \boxed{\frac{-7}{25}}$

12. Given $\cos\alpha = \dfrac{\sqrt{3}}{2}$ and α is in quadrant IV, find $\sin\dfrac{\alpha}{2}$.

$1-\sqrt{3}-2$ sin is neg.

$= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \boxed{-\frac{\sqrt{2-\sqrt{3}}}{2}}$

13. Given $\sin\alpha = -\dfrac{4}{5}$ and α is in quadrant IV, find $\cos(2\alpha)$.

3-4-5 cos is pos

$\boxed{\frac{-7}{25}}$

14. Given $\cos\alpha = -\dfrac{5}{13}$ and α is in quadrant II, find $\sin\dfrac{\alpha}{2}$.

5-12-13 sin is pos.

$= \sqrt{\frac{1-\frac{-5}{13}}{2}} = \sqrt{\frac{\frac{18}{13}}{2}} = \sqrt{\frac{18}{13}\cdot\frac{1}{2}} = \sqrt{\frac{9}{13}} = \boxed{\frac{3\sqrt{13}}{13}}$

Objective: Use knowledge of inverse trig functions to find exact values of each expression.

Find the exact value of each expression. Do not use a calculator.

15. $\sin^{-1}\dfrac{-\sqrt{2}}{2} = \dfrac{-\pi}{4}$

16. $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) = \dfrac{5\pi}{6}$

17. $\sin\left(\tan^{-1}\dfrac{3}{4}\right) = \dfrac{3}{5}$

3-4-5

18. $\sin^{-1}\left(\cos\dfrac{2\pi}{3}\right) = \dfrac{-\pi}{6}$

$-1/2$

19. $\tan\left[\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)\right] = \dfrac{-\sqrt{3}}{3}$

$-\pi/3$

20. $\sin\left[2\cos^{-1}\left(-\dfrac{3}{5}\right)\right] = \dfrac{24}{25}$

3-4-5

Solve each trig equation on the interval $[0, 2\pi]$

21. $2\sin x + \sqrt{2} = 0$

$\sin x = \dfrac{-\sqrt{2}}{2}$ $\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

22. $2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{\sqrt{2}}{2}$ $\boxed{x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}}$

23. $3\cos^2 x - 4 = 0$

$\csc^2 x = \frac{4}{3}$

$\csc x = \frac{2\sqrt{3}}{3}$ $x =$

So $\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

24. $2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\sin x = -1$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$ $\boxed{x = \frac{3\pi}{2}}$

25. $(2\cos x - \sqrt{3})(2\sin x - 1) = 0$

$\cos x = \frac{\sqrt{3}}{2}$ $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

26. $\tan x + \sqrt{3} = 0$

$\tan x = -\sqrt{3}$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$

27. $\sin^2 x + \sin x = 0$

$\sin x(\sin x + 1) = 0$

$\sin x = 0$ $x = 0, \pi$ $\sin x = -1$ $x = \frac{3\pi}{2}$

28. $\tan x \cos x - \cos x = 0$

$\cos x(\tan x - 1) = 0$

$\cos x = 0$ $x = \pi/2, 3\pi/2$ $\tan x = 1$ $x = \frac{\pi}{4}, \frac{5\pi}{4}$

Objective: Students will utilize the unit circle to determine: the location of the terminal ray of an angle, the exact value of a trigonometric function, the quadrant given specifications, the sign of the value of a function.

29. Find the value of the cos $-5\pi/3$. $= \cos\frac{\pi}{3} = \frac{1}{2}$

30. Find the exact value of tan $4\pi/3$. $= \sqrt{3}$

31. Identify the quadrant if the $\csc\theta < 0$ and the $\tan\theta < 0$. Q4

(csc neg, tan neg)

32. The value of sin $8\pi/7$ will be: negative

Q3

Objective: Utilize the Unit Circle to solve trig equations on the interval $[0, 2\pi)$:)

33. Solve: $2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$ $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

34. Give the solutions for $4\tan^2\theta = 4$

$\tan^2\theta = 1$ $\tan\theta = \pm 1$ $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

35. Solve $\csc x - 2 = 0$ for $[0, 2\pi]$

$\csc x = 2$

so $\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

36. Give the solutions for $2\sin\theta\cos\theta = -\cos\theta$

$2\sin\theta\cos\theta + \cos\theta = 0$

$\cos\theta(2\sin\theta + 1) = 0$

$\cos\theta = 0$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\sin\theta + 1 = 0$ $\sin\theta = -\frac{1}{2}$ $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

Objective: Students will graph trigonometric functions and identify period, amplitude, phase shift, increasing/decreasing intervals, and max and minimum values, and asymptotes.

37. Give the **maximum** value of one period of $f(x) = \frac{1}{2}\cos x$.

max = 1

38. Give the **minimum** value of one period of $f(x) = \sin x - 6$

min = -7

39. **Where** does the maximum value occur for $y = \sin(x + \pi)$ normal sin has max @ $\frac{\pi}{2}$.

move it π to left, max is @ $-\frac{\pi}{2}$

40. What are the **x-intercept(s)** of the graph $y = -2\sin(x) + 2$?

$0 = -2\sin x + 2$

$-2 = -2\sin x$

$1 = \sin x$ so $x = \frac{\pi}{2}$

41. What are the **x-intercepts** of $y = 3\cos(2x)$?

period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Key points @ $\frac{\pi}{4}$. so x-ints @ $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, etc.

42. Give the **amplitude** of $y = -4\sin(\pi x)$.

$a = 4$

43. Give the **period** of $y = 2\sin(\frac{x}{2})$.

period $= \frac{2\pi}{b} = \frac{2\pi}{1/2} = 4\pi$