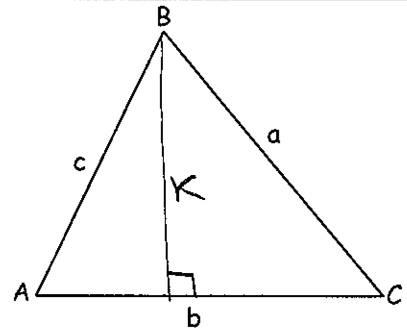
Precalculus Name Key

6.1 Notes: Law of Sines–*Day 1 (AAS & ASA)*

Consider oblique $\triangle ABC$ for each of the following. Oblique triangles are triangles that have no right angles.



1. In $\triangle ABC$, sketch an altitude from vertex B. Label the altitude k.

2. The altitude creates two right triangles. Notice that $\angle A$ is contained in one of the right triangles and $\angle C$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$ and one involving $\sin C$.

$$\sin A = \frac{K}{c} \qquad \sin C = \frac{K}{a}$$

3. Notice that each of the equations in **Question 2** involves k. Solve each equation for k (get k by itself on one side of the equal sign).

$$K = c\sin A \qquad K = a\sin C$$

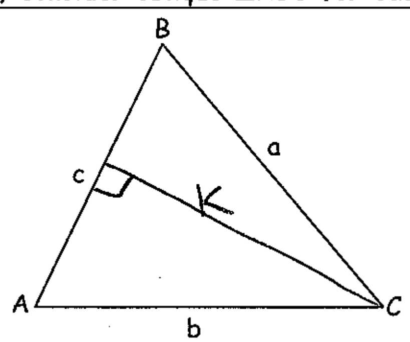
4. Since both equations in **Question 3** are equal to k, they can be set equal to each other. Set the equations equal to each other to form a new equation.

$$c\sin A = a\sin C$$

5. Notice that the equation in **Question 4** no longer involves k. Write an equation equivalent to the equation in **Question 4** by regrouping a with $\sin A$ and c with $\sin C$.

$$\frac{\cancel{c}\sin A}{\cancel{c}\,a} = \frac{\cancel{a}\sin C}{\cancel{a}\,c} \qquad \frac{\sin A}{a} = \frac{\sin C}{c}$$

Again, consider oblique $\triangle ABC$ for each of the following.



6. In $\triangle ABC$, sketch an altitude from vertex C. Label the altitude k.

7. The altitude creates two right triangles. Notice that $\angle A$ is contained in one of the right triangles and $\angle B$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$ and one involving $\sin B$.

$$\sin A = \frac{K}{b} \qquad \sin B = \frac{K}{a}$$

8. Notice that each of the equations in **Question 7** involves k. Solve each equation for k (get k by itself on one side of the equal sign).

$$K = b\sin A \qquad K = a\sin B$$

9. Since both equations in **Question 8** are equal to k, they can be set equal to each other. Set the equations equal to each other to form a new equation.

$$b\sin A = a\sin B$$

10. Notice that the equation in **Question 9** no longer involves k. Write an equation equivalent to the equation in **Question 9** by regrouping a with $\sin A$ and b with $\sin B$.

$$\frac{\cancel{b}\sin A}{\cancel{b}\,a} = \frac{\cancel{a}\sin B}{\cancel{a}\,b} \qquad \frac{\sin A}{a} = \frac{\sin B}{b}$$

11. The two equations from Questions 5 and 10 on the front are part of the law of sines formula. Combine these formulas to create one equation.

$$\frac{\sin A}{a} = \frac{\sin C}{c} \qquad \frac{\sin A}{a} = \frac{\sin B}{b} \rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

* transitive property

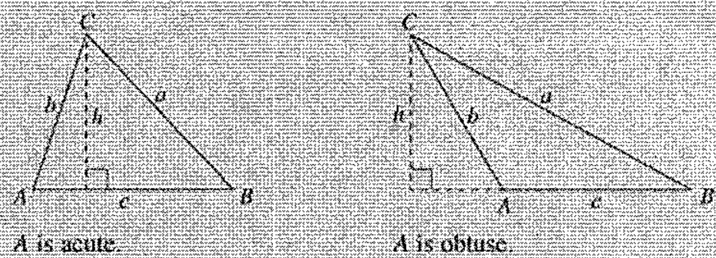
Law of Sines (See the proof on page 468.)

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Reciprocal

Oblique Triangles



A is acute   A is obtuse

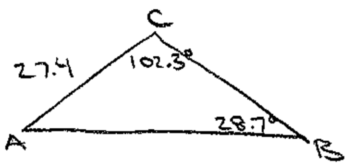
The law of sines can be used to solve a triangle if the following information is known:

- Two angles and any side (AAS or ASA)
- Two sides and an angle opposite one of them (SSA). We will study this case after break.

Examples: Use the law of sines to solve the triangle. Round answers to the nearest hundredth.

1. $C = 102.3°, B = 28.7°,$ and $b = 27.4$ feet

(AAS)



$A = 180° - 102.3° - 28.7°$

$A = 49°$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{27.4}{\sin 28.7°} = \frac{c}{\sin 102.3°}$$

$$c = \sin 102.3 \left(\frac{27.4}{\sin 28.7}\right)$$

$c = 55.75$ ft.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

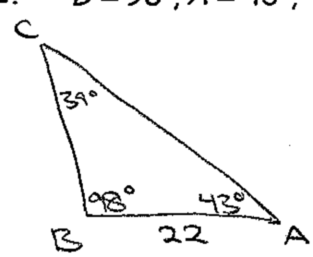
$$\frac{27.4}{\sin 28.7°} = \frac{a}{\sin 49°}$$

$$a = \sin 49 \left(\frac{27.4}{\sin 28.7}\right)$$

$a = 43.06$ ft.

2. $B = 98°, A = 43°,$ and $c = 22$ feet

(ASA)



$C = 180° - 98° - 43°$

$C = 39°$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{22}{\sin 39°} = \frac{a}{\sin 43°}$$

$$a = \sin 43° \left(\frac{22}{\sin 39°}\right)$$

$a = 23.84$ ft.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{22}{\sin 39°} = \frac{b}{\sin 98°}$$

$$b = \sin 98° \left(\frac{22}{\sin 39°}\right)$$

$b = 34.62$ ft.