Precalculus Name Key

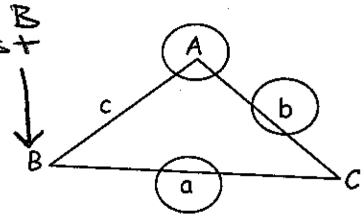
6.1 Notes: Law of Sines–Day 2 (SSA)

The Ambiguous Case: SSA

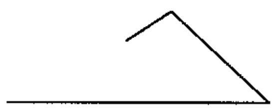
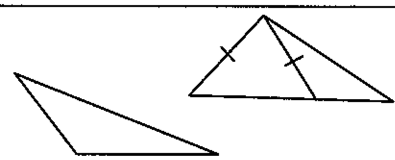
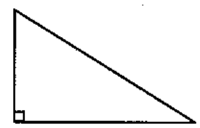
Two angles and one side determine a unique triangle. However, if two sides and one opposite angle are given, three possible situations occur:

- No triangle exists
- One such triangle exists
- Two distinct triangles exist

Find B first

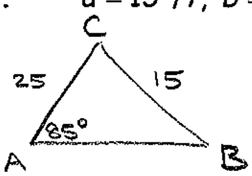


Given the measure of one angle and two consecutive sides (SSA), you may use the following table to determine how many solutions there are for the triangle. *Note that this table represents a general example and angle B is simply the angle that can be found first once the law of sines is set up.*

sin B > 1	0 < sin B < 1	sin B = 1
NO SUCH TRIANGLE EXISTS	1 SOLUTION IF $m\angle B \le m\angle A$ 2 SOLUTIONS IF $m\angle B > m\angle A$	ONE SOLUTION
		
The sine of an angle cannot be greater than one. This means that one of the given sides is much too long for the triangle to form.	For one solution to occur, the angle that you can find first must be smaller than the given angle. For 2 solutions to occur, the angle you can find first must be larger than the given angle. The second case is found by finding the *supplement* to the angle that you can find first.	If the sine of the angle you can find first is exactly 1, then the trianlge must be a right triangle. You may use SOHCAHTOA, Pythagorean Theorem, or possibly your special right triangle ratios to solve this triangle.

Solve the triangle. If two solutions exist, find both. Round to the nearest hundredth.

1. $a = 15$ ft, $b = 25$ ft, $A = 85°$ SSA

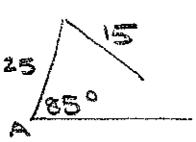


$$\frac{\sin B}{25} = \frac{\sin 85°}{15}$$

$$\sin B = 25\left(\frac{\sin 85°}{15}\right)$$

$$B = \sin^{-1}\left(25 \cdot \frac{\sin 85°}{15}\right)$$

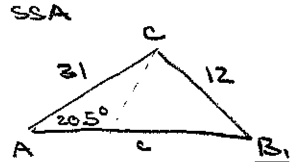
$\boxed{\text{No triangle exists}}$



* Doesn't close

2. $a = 12$ meters, $b = 31$ meters, $A = 20.5°$

SSA



$$\frac{\sin B}{31} = \frac{\sin 20.5°}{12}$$

$\boxed{B_1 = 64.78°}$

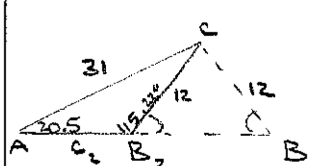
* Two Δs exist!

$\boxed{C_1 = 94.72°}$

$$\frac{c}{\sin 94.72°} = \frac{12}{\sin 20.5°}$$

$\boxed{c_1 = 34.15 \text{ m.}}$

$\boxed{C_2 = 44.28°}$



$$\frac{c}{\sin 44.28°} = \frac{12}{\sin 20.5°}$$

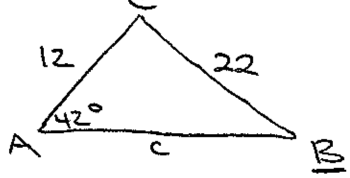
$\boxed{c_2 = 23.92 \text{ m}}$

$B_2 = 180° - 64.78°$

$\boxed{B_2 = 115.22°}$

Continue to solve the triangle. If two solutions exist, find both. Round to the nearest hundredth.

3. $a = 22$ *inches*, $b = 12$ *inches*, $A = 42°$ SSA



$$\frac{\sin B}{12} = \frac{\sin 42°}{22}$$

$B = 21.41°$

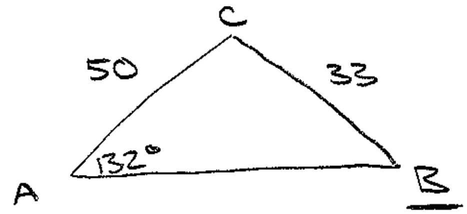
* One triangle exists!

$C = 116.59°$

$$\frac{c}{\sin 116.59°} = \frac{22}{\sin 42°}$$

$c = 29.40$ in

4. $b = 50$, $a = 33$, $A = 132°$ SSA

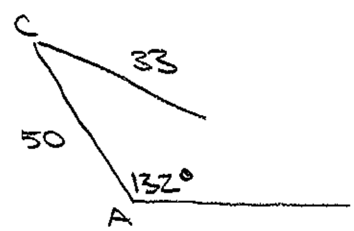


$$\frac{\sin B}{50} = \frac{\sin 132°}{33}$$

$B = \sin^{-1}(1.126)$

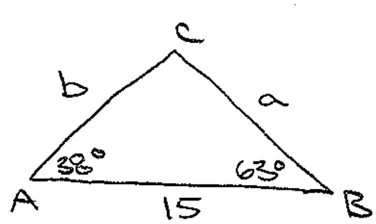
Not in Domain

No triangle exists



5. $A = 38°, B = 63°, c = 15$ ASA

* One triangle



$C = 79°$

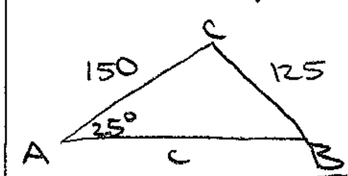
$$\frac{a}{\sin 38°} = \frac{15}{\sin 79°}$$

$a = 9.41$ u.

$$\frac{b}{\sin 63°} = \frac{15}{\sin 79°}$$

$b = 13.62$ u.

6. $a = 125, A = 25°, b = 150$ SSA



$$\frac{\sin B}{150} = \frac{\sin 25}{125}$$

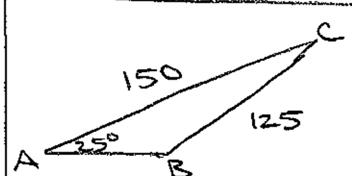
$B_1 = 30.47°$

* 2 triangles!

$C_1 = 124.53°$

$$\frac{c}{\sin 124.53°} = \frac{125}{\sin 25°}$$

$c_1 = 243.67$ u.



$B = 180° - 30.47°$

$B_2 = 149.53°$

$C_2 = 5.47°$

$$\frac{c}{\sin 5.47°} = \frac{125}{\sin 25°}$$

$c_2 = 28.19$ u