Name __Key__

Divide:

1. $(4x^2 + 7x - 3) \div (x + 2)$

$$-2 \begin{array}{|ccc} 4 & 7 & -3 \\ & -8 & 2 \\ \hline \end{array}$$
$$4 \quad -1 \mid -1$$

$$\boxed{4x - 1 - \frac{1}{x+2}}$$

3. $(2x^2 + 7x - 39) \div (2x - 7)$

$$\begin{array}{r} x + 7 \\ 2x-7 \overline{\smash{\big)}\, 2x^2 + 7x - 39} \\ -\ 2x^2 - 7x \\ \hline 14x - 39 \\ -\ 14x - 49 \\ \hline 10 \end{array}$$

$$\boxed{x + 7 + \frac{10}{2x-7}}$$

2. $(8x^2 + x^3 - 3x - 5) \div (x - 3)$

$(x^3 + 8x^2 - 3x - 5)$

$$3 \begin{array}{|cccc} 1 & 8 & -3 & -5 \\ & 3 & 33 & 90 \\ \hline \end{array}$$
$$1 \quad 11 \quad 30 \mid 85$$

$$\boxed{x^2 + 11x + 30 + \frac{85}{x-3}}$$

4. $(2x^4 + 1 - 2x - x^3) \div (2x - 1)$

$$\begin{array}{r} x^3 + 0x^2 + 0x - 1 \\ 2x-1 \overline{\smash{\big)}\, 2x^4 - x^3 + 0x^2 - 2x + 1} \\ -\ 2x^4 - x^3 \\ \hline 0x^3 + 0x^2 \\ -\ 0x^3 + 0x^2 \\ \hline 0x^2 - x \end{array}$$

$$\boxed{x^3 - 1}$$

$$\begin{array}{r} 0x^2 - 2x \\ -\ 0x + 0 \\ \hline -2x + 1 \\ -\ -2x + 1 \\ \hline 0 \end{array}$$

Determine if the given binomial is a factor of the polynomial. Justify your answer.

5. $(x + 2); (x^4 - 2x^3 + 2x^2 + x - 10)$ $\boxed{\text{NO}}$

$(-2)^4 - 2(-2)^3 + 2(-2)^2 + (-2) - 10 = 28$

6. $(x - 5); (4x^3 - 8x + 7)$ $\boxed{\text{NO}}$

$4(5)^3 - 8(5) + 7 = 467$

For each of the following polynomials, determine each of the following. Then sketch the polynomial.

A. End behavior B. x-intercepts C. Multiplicity D. y-intercept

7. $f(x) = -x^3 - 3x^2$

$x \to \infty, y \to -\infty$

A. $x \to -\infty, y \to \infty$

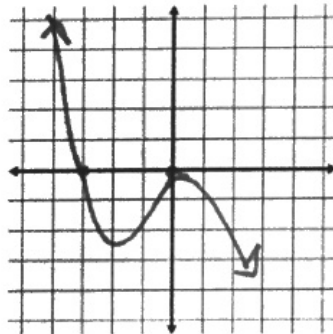$-x^2(x+3) = 0$

B. $x = 0 \quad x = -3$

$x = 0$ mult. 2

C. $x = -3$ mult. 1

D. $(0, 0)$

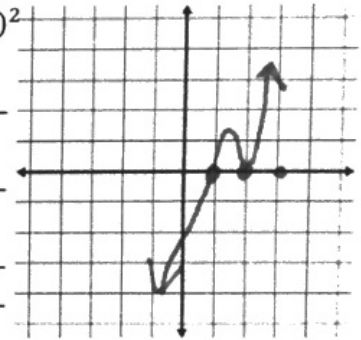**8.** $f(x) = 3(x - 1)(x - 2)^2$

$x \to \infty, y \to \infty$

A. $x \to -\infty, y \to \infty$

$x = 1 \quad x = 2$

B.

$x = 1$ mult. 1

C. $x = 2$ mult. 2

D. $(0, -12)$

9. $f(x) = -2(x + 3)^2(x + 1)^2$

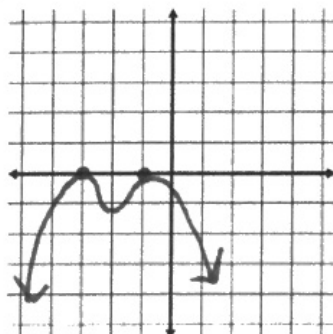$x \to \infty, y \to -\infty$

A. $x \to -\infty, y \to -\infty$

$x = -3$

B. $x = -1$

$x = -3$ mult 2

C. $x = -1$ mult 2

D. $(0, -18)$

10. $f(x) = 3x^4 - 3x^3 - 3x^2 + 3x$

$x \to \infty, y \to \infty$

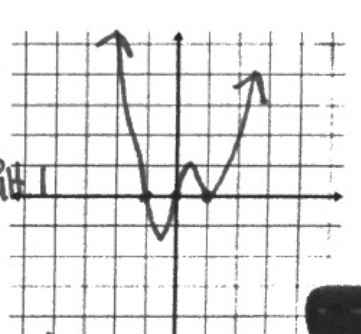A. $x \to -\infty, y \to \infty$

$x = 1 \quad x = 0$

B. $x = -1$

$x = 1$ mult. 1 $x = 0$ mult 1

C. $x = 2$ mult. 2

D. $(0, 0)$

$f(x) = 3x(x^3 - x^2 - x + 1)$

$f(x) = 3x(x^2(x-1) - 1(x-1))$

$f(x) = 3x(x^2 - 1)(x - 1)$

$f(x) = 3x(x + 1)(x - 1)(x - 1)$

$f(x) = 3x(x + 1)(x - 1)^2$

Solve each polynomial.

11. $x^4 - 5x^2 - 36 = 0$ $\boxed{x = \pm 2i, \pm 3}$

$(x^2 + 4)(x^2 - 9) = 0$

$x^2 + 4 = 0$ $x^2 - 9 = 0$

$x^2 = -4$ $x^2 = 9$

$x = \pm\sqrt{-4}$ $x = \pm\sqrt{9}$

$x = \pm 2i$ $x = \pm 3$

12. $x^3 - 2x^2 + 3x - 6 = 0$ $\boxed{x = 2, \pm i\sqrt{3}}$

$x^2(x-2) + 3(x-2) = 0$

$(x^2 + 3)(x - 2) = 0$

$x^2 + 3 = 0$ $x - 2 = 0$

$x^2 = -3$ $x = 2$

$x = \pm\sqrt{-3} = \pm i\sqrt{3}$

13. $x^3 - 125 = 0$

$x^3 - 5^3 = 0$

$(x - 5)(x^2 - 5x + 25) = 0$

$\boxed{x = 5}$ $x = \dfrac{5 \pm \sqrt{25 - 4(1)(25)}}{2(1)}$

$x = \dfrac{5 \pm \sqrt{-75}}{2}$

$x = \dfrac{5 \pm \sqrt{25}\sqrt{3}}{2}$

$\boxed{x = \dfrac{5 \pm 5i\sqrt{3}}{2}}$

14. $x^3 + 8x^2 + 22x + 20 = 0$

zero at $x = -2$

$$-2 \,\big|\; 1 \quad 8 \quad 22 \quad 20$$
$$\; -2 \;\; -12 \;\; -20$$
$$\overline{1 \quad 6 \quad 10 \quad 0}$$

$(x + 2)(x^2 + 6x + 10) = 0$

$\boxed{x = -2}$ $x = \dfrac{-6 \pm \sqrt{36 - 4(1)(10)}}{2(1)}$

$\boxed{x = -3 \pm i}$

$x = \dfrac{-6 \pm \sqrt{-4}}{2} = \dfrac{-6 \pm 2i}{2} = -3 \pm i$

Write a polynomial given the following solutions.

15. $x = 4, multiplicity\ 3; x = 0$

$y = (x-4)^3 \ (x)$

$y = (x-4)(x-4)(x-4)(x)$

$y = (x^2 - 8x + 16)(x^2 - 4x)$

$y = x^4 - 8x^3 + 16x - 4x^3 + 32x^2 - 64x$

$\boxed{y = x^4 - 12x^3 + 32x^2 - 48}$

16. $x = \sqrt{5}, x = -8, multiplicity\ 2$

$y = (x - \sqrt{5})(x + \sqrt{5})(x + 8)^2$

$y = (x^2 - 5)(x^2 + 16x + 64)$

$y = x^4 + 16x^3 + 64x^2 - 5x^2 - 80x - 320$

~~$y = x^4 + 16x^3$~~

$\boxed{y = x^4 + 16x^3 + 59x^2 - 80x - 320}$

17. $x = 6, x = 1 \pm 4i$

$y = (x-6)(x - (1 + 4i))(x + (1 - 4i))$

$y = (x-6)((x-1) - 4i)((x-1) + 4i)$

$y = (x-6)((x-1)^2 - (4i)^2)$

$y = (x-6)((x-1)(x-1) + 16)$

$y = (x-6)(x^2 - 2x + 1 + 16)$

$y = (x-6)(x^2 - 2x + 17)$

$y = x^3 - 2x^2 + 17x - 6x^2 + 12x - 102$

$\boxed{y = x^3 - 8x^2 + 29x + 102}$

18. $x = 5i, x = 1, x = 0$

$y = (x - 5i)(x + 5i)(x - 1)(x)$
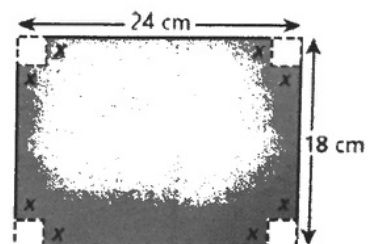
$y = (x^2 + 25)(x^2 - x)$

$\boxed{y = x^4 - x^3 + 25x^2 - 25x}$

Applications:

19. A picture frame with dimensions $10\ in \times 14\ in$ has a width $x\ cm$ wide on each side. If the picture displayed in the frame has an area of $77\ in^2$, what is the value of x?

20. A cake pan is made by cutting four squares from a 18 cm by 24 cm piece of tin and folding up the sides as shown.

    a. If the volume of the cake pan is 640 cm², find the dimensions of the cake pan?

    b. Bonus: What value of x would give the cake pan the largest volume?