Pre-Calculus

Name: ISEY

Notes 1.2 (Part 2)

Goal #1: Students will be able to identify local and absolute extrema.
Goal #2: Students will be able to use interval notation to state increasing, decreasing, and constant intervals.
Goal #3: Students will be able to determine the symmetry of a function and classify it as “even,” “odd,” or “neither.“
Goal #4: Students will be able to determine the boundedness of a function.

Local & Absolute Extrema

DEFINITION Local and Absolute Extrema

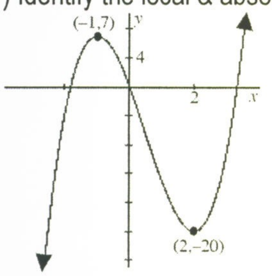
A **local maximum** of a function f is a value $f(c)$ that is greater than or equal to all range values of f on some open interval containing c. If $f(c)$ is greater than or equal to all range values of f, then $f(c)$ is the **maximum** (or **absolute maximum**) value of f.

A **local minimum** of a function f is a value $f(c)$ that is less than or equal to all range values of f on some open interval containing c. If $f(c)$ is less than or equal to all range values of f, then $f(c)$ is the **minimum** (or **absolute minimum**) value of f.

Local extrema are also called **relative extrema**.

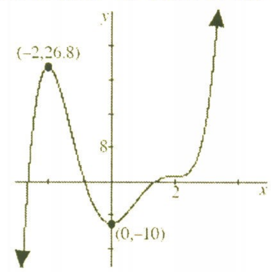
Ex1) Identify the local & absolute maximums & minimums for each of the functions below:

a)



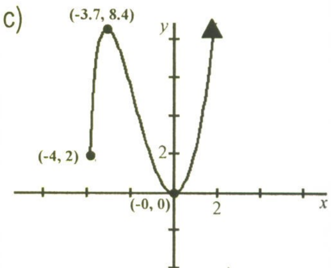
Local minimum: -20
Local maximum: 7
Absolute minimum: None
Absolute maximum: None

b)



Local minimum: -10
Local maximum: 26.8
Absolute minimum: None
Absolute maximum: None

c)



Local minimum: 2 & 0
Local maximum: 8.4
Absolute minimum: 0
Absolute maximum: None

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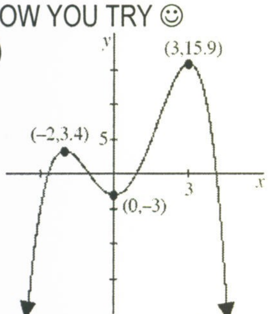
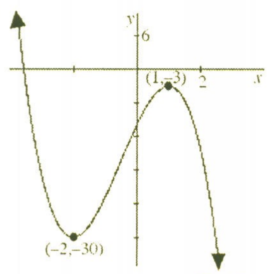
d)



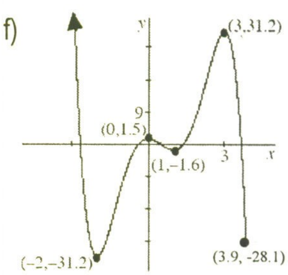
Local minimum: -3
Local maximum: 3.4 & 15.9
Absolute minimum: None
Absolute maximum: 15.9

e)



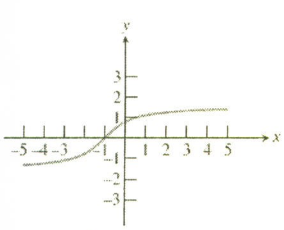
Local minimum: -30
Local maximum: -3
Absolute minimum: None
Absolute maximum: None

f)

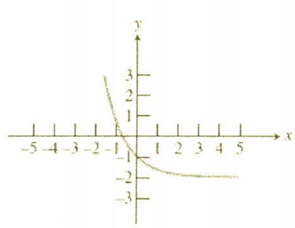


Local minimum: -31.2 & -1.6 & -28.1
Local maximum: 1.5 & 31.2
Absolute minimum: -31.2
Absolute maximum: None

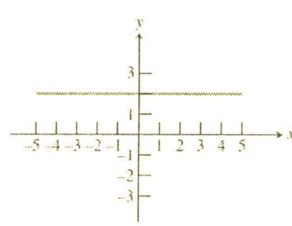
Increasing & Decreasing Interval



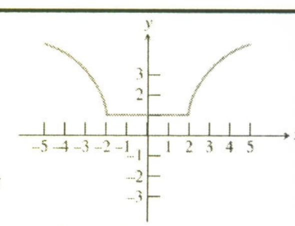
Increasing



Decreasing



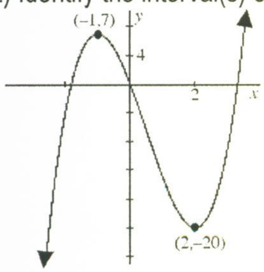
Constant



Decreasing on $(-\infty, -2]$
Constant on $[-2, 2]$
Increasing on $[2, \infty)$

Ex2) Identify the interval(s) over which the following functions are increasing & decreasing:

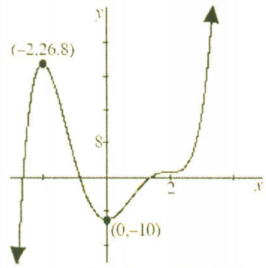
a)



Increasing: $(-\infty,-1)\cup(2,\infty)$

Decreasing: $(-1,2)$

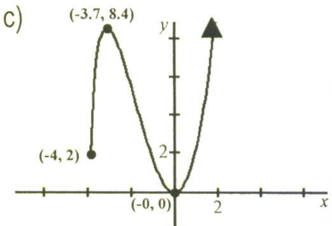
b)



Increasing: $(-\infty,-2)\cup(0,\infty)$

Decreasing: $(-2,0)$

c)



Increasing: $[-4,-3.7)\cup(0,\infty)$

Decreasing: $(-3.7,0)$

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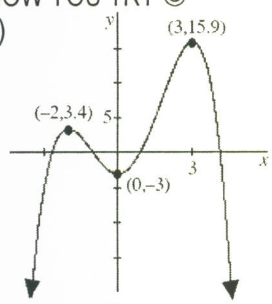
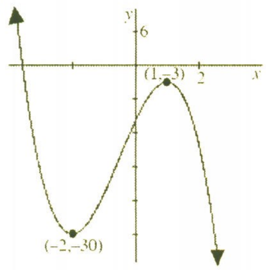
d)



Increasing: $(-\infty,-2)\cup(0,3)$

Decreasing: $(-2,0)\cup(3,\infty)$

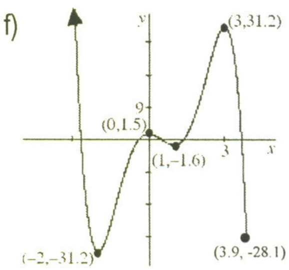
e)



Increasing: $(-2,1)$

Decreasing: $(-\infty,-2)\cup(1,\infty)$

f)



Increasing: $(-2,0)\cup(1,3)$

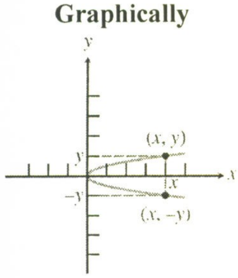
Decreasing: $(-\infty,-2)\cup(0,1)\cup(3,3.9]$

Symmetry

In the graphical sense, the word "symmetry" in mathematics carries essentially the same meaning as it does in art: The picture (in this case, the graph) "looks the same" when viewed in more than one way. The interesting thing about mathematical symmetry is that it can be characterized **ALGEBRAICALLY** as well. We will be looking at three particular types of symmetry, each of which can be spotted easily from a graph, or an algebraic formula.

Symmetry With Respect the x-axis.

Graphically

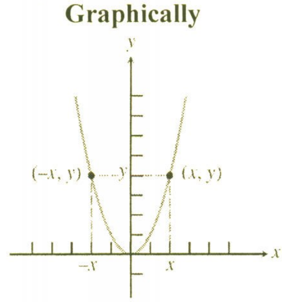


Algebraically

Graphs with this kind of symmetry are not functions (except the zero function), but we can say that $(x, -y)$ is on the graph whenever (x, y) is on the graph.

Symmetry With Respect the y-axis. "EVEN"

Graphically



Algebraically

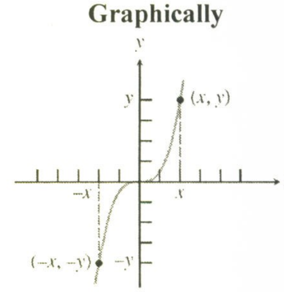
For all x in the domain of f,

$$f(-x) = f(x)$$

Functions with this property (for example, x^n, n even) are **even** functions.

Symmetry With Respect the origin. "ODD"

Graphically



Algebraically

For all x in the domain of f,

$$f(-x) = -f(x).$$

Functions with this property (for example, x^n, n odd) are **odd** functions.

Ex3) Determine algebraically if each of the following functions is even, odd, or neither.

a) $f(x) = x^2 - 3$

$f(-x) = (-x)^2 - 3$

$f(-x) = x^2 - 3$

$f(x) = f(-x) \therefore$ EVEN

b) $g(x) = x^2 - 2x - 2$

$g(-x) = (-x)^2 - 2(-x) - 2$

$g(-x) = x^2 + 2x - 2$

$g(x) \neq g(-x) \therefore$ NOT EVEN

$-g(x) = -(x^2 - 2x - 2)$

$-g(x) = -x^2 + 2x + 2$

$-g(x) \neq g(-x) \therefore$ NOT ODD

NEITHER

c) $h(x) = \dfrac{x^3}{4 - x^2}$

$h(-x) = \dfrac{(-x)^3}{4-(-x)^2} = \dfrac{-x^3}{4-x^2}$

$-h(x) = -\dfrac{x^3}{4-x^2}$

$h(-x) = -h(x) \therefore$ ODD

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Square root
Not 4th ROOT

d) $f(x) = -2x^4\sqrt{x+3}$

$f(-x) = -2(-x)^4\sqrt{(-x)+3}$

$f(-x) = -2x^4\sqrt{-x+3}$

$f(-x) \neq f(x) \therefore$ NOT EVEN

$-f(x) = -(-2x^4\sqrt{x+3})$

$-f(x) = 2x^4\sqrt{x+3}$

$-f(x) \neq f(-x) \therefore$ NOT ODD

NEITHER

e) $g(x) = 7x^5 - 4x^3 + 11x$

$g(-x) = 7(-x)^5 - 4(-x)^3 + 11(-x)$

$g(-x) = -7x^5 + 4x^3 - 11x$

$g(-x) \neq g(x) \therefore$ NOT EVEN

$-g(x) = -(7x^5 - 4x^3 + 11x)$

$-g(x) = -7x^5 + 4x^3 - 11x$

$g(-x) = -g(x) \therefore$ ODD

f) $h(x) = \dfrac{x^3}{4x - x^5}$

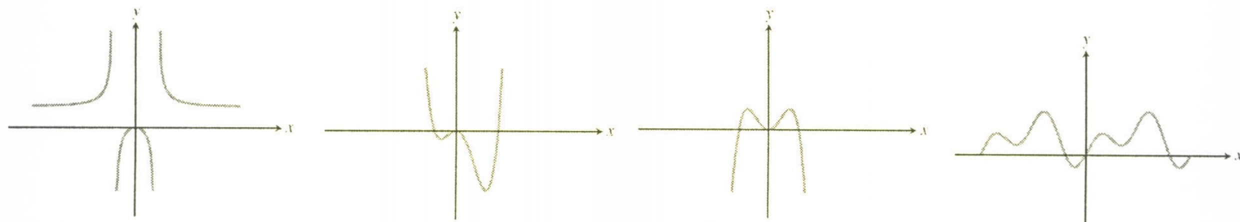
$h(-x) = \dfrac{(-x)^3}{4(-x) - (-x)^5}$

$h(-x) = \dfrac{-x^3}{-4x + x^5} = \dfrac{-1 \cdot x^3}{-1(4x - x^5)}$

$h(-x) = h(x) \therefore$ EVEN

Boundedness

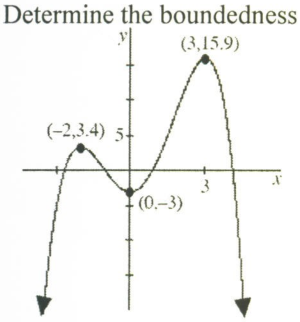
- A function *f* is "***BOUNDED BELOW***" if there is some number *b* that is less than or equal to every number in the range of *f*.
- A function *f* is "***BOUNDED ABOVE***" if there is some number *B* that is greater than or equal to every number in the range of *f*.
- A function *f* is "***BOUNDED***" if it is bounded both above and below.
- A function *f* is "***UNBOUNDED***" if it is bounded both above and below.

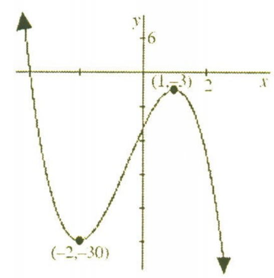


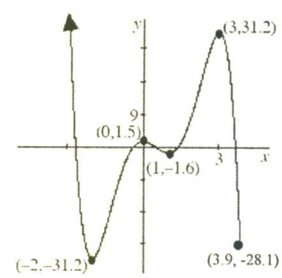
Not bounded above / Not bounded below — Not bounded above / Bounded below — Bounded above / Not bounded below — Bounded

Ex4) Determine the boundedness of the functions below:

a)







Boundedness: BOUNDED ABOVE Boundedness: NOT BOUNDED Boundedness: BOUNDED BELOW

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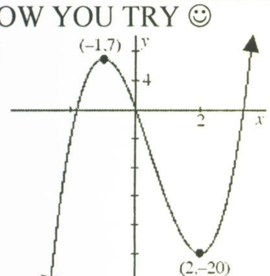
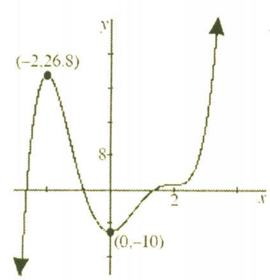
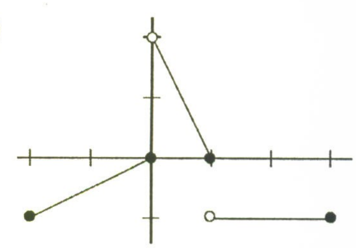
d)



e)



f)



Boundedness: NOT BOUNDED Boundedness: NOT BOUNDED Boundedness: BOUNDED