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Transformations of Graphs

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This is a summary notes about transformation of graphs.
The primary focus will be on learning to understand the
kind of information that these graphs convey.
It will be helpful to be familiar with the general shapes
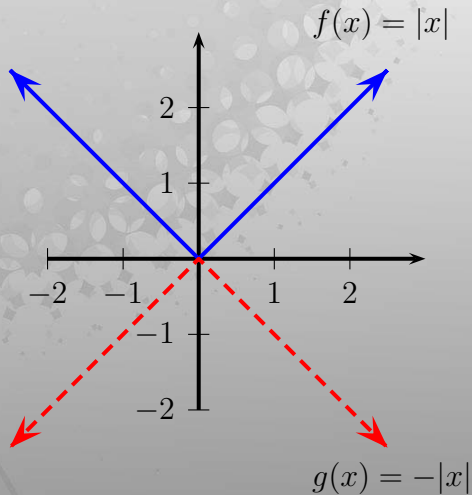of these graphs without resorting to point-plotting.

Reflection

The graph of $g(x) = -\sqrt{x}$ is a mirror image of the graph of $f(x) = \sqrt{x}$.
We say that the graphs of $f(x)$ and $g(x)$ are reflections of one another about
the x-axis.
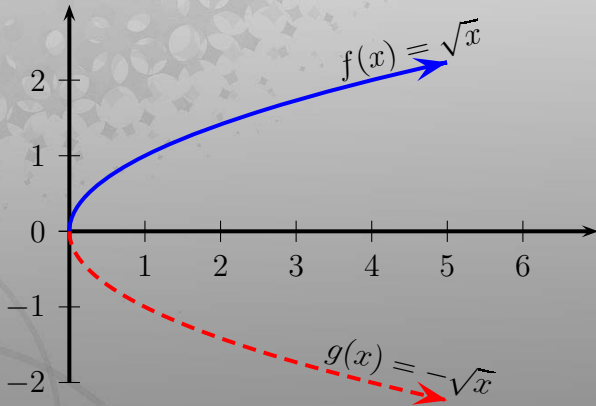
The graph of $g(x) = -x^2$ is obtained by reflecting the graph of $f(x) = x^2$ in the x-axis.
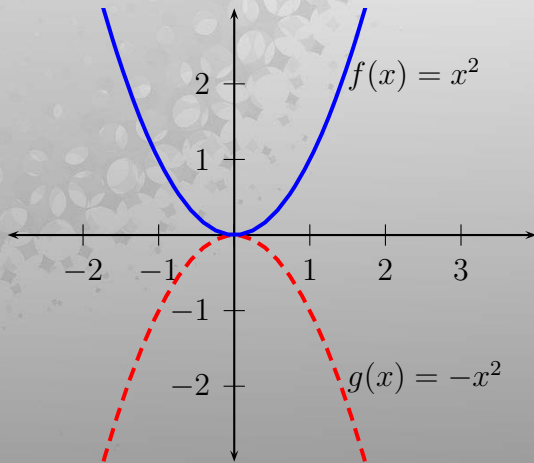
The graph of $g(x) = -|x|$ is a reflection in the x-axis of the graph of $f(x) = |x|$.
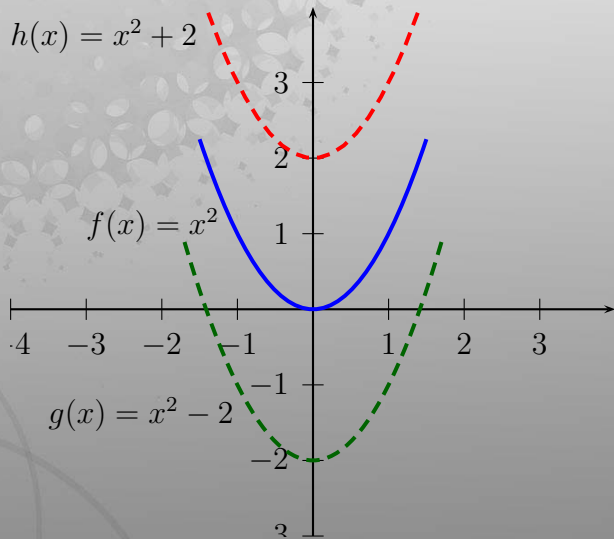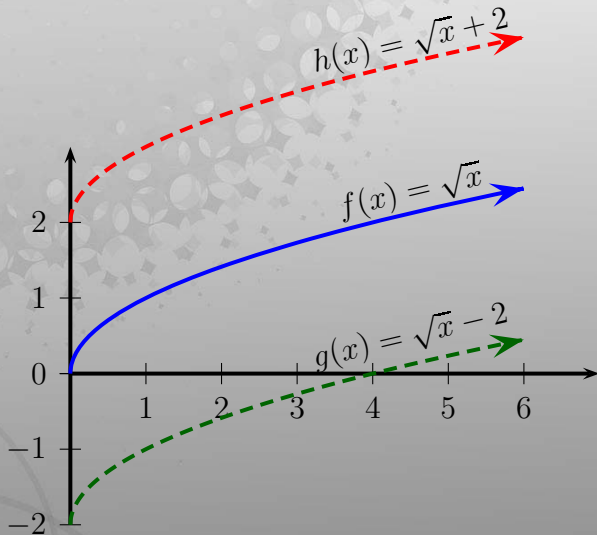
Translating

Translating Upward or Downward

Equation	$y = f(x) + c$ with $c > o$	$y = f(x) - c$ with $c > o$
Effect on graph	The graph of $f(x)$ is shifted vertically upward a distance c	The graph of $f(x)$ is shifted vertically downward a distance c

The graph of $h(x) = x^2 + 2$ is an upward translation of the graph of $f(x) = x^2$, and the graph of $g(x) = x^2 - 2$ is a downward translation of the graph of $f(x) = x^2$.

The graph of $h(x) = \sqrt{x} + 2$ is an upward translation of the graph of $f(x) = \sqrt{x}$, and the graph of $g(x) = \sqrt{x} - 2$ is a downward translation of the graph of $f(x) = \sqrt{x}$.
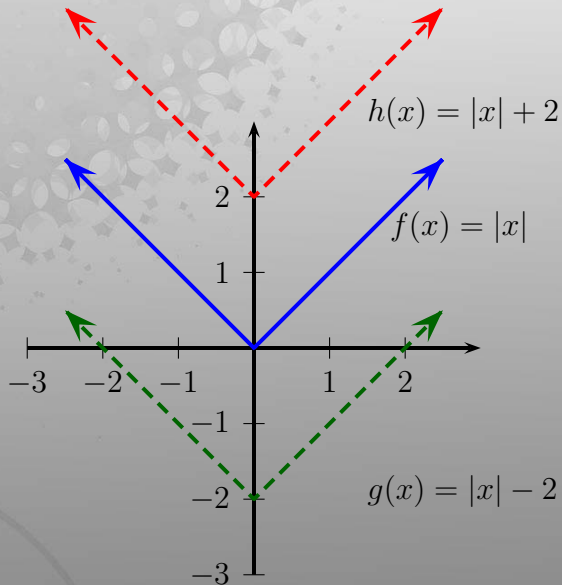
The graph of $h(x) = |x| + 2$ is an upward translation of the graph of $f(x) = |x|$, and the graph of $g(x) = |x| - 2$ is a downward translation of the graph of $f(x) = |x|$.

$h(x) = |x| + 2$

$f(x) = |x|$

$g(x) = |x| - 2$

Translating to the Right or Left

Equation	$y = f(x - c)$ with $c > o$	$y = f(x + c)$ with $c > o$
Effect on graph	The graph of $f(x)$ is shifted horizontally to the right a distance c.	The graph of $f(x)$ is shifted horizontally to the left a distance c.

Consider the graphs of $f(x) = x^2$, $g(x) = (x-2)^2$, and $h(x) = (x+2)^2$. Every point on the graph of g is exactly two units to the right of a corresponding point on the graph of f.

Also every point of the graph h is exactly two units to the left of of a corresponding point on the graph of f.
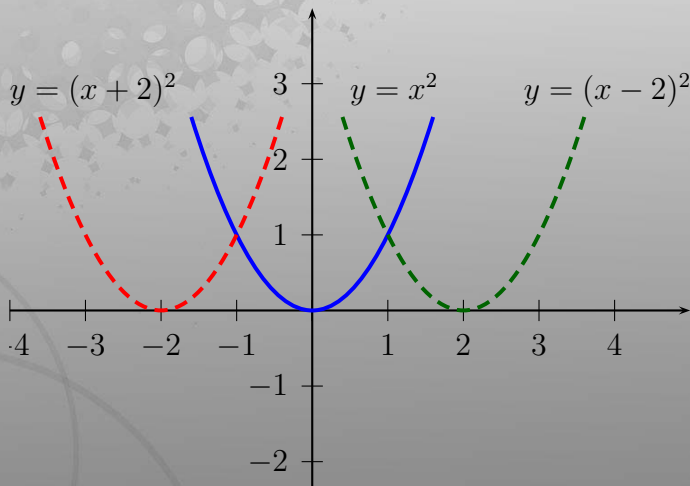
Consider the graphs of $f(x) = |x|$, $g(x) = |x - 2|$, and $h(x) = |x + 2|$.
Every point on the graph of g is exactly two units to the right of a corresponding point on the graph of f.
Also every point of the graph h is exactly two units to the left of of a corresponding point on the graph of f.
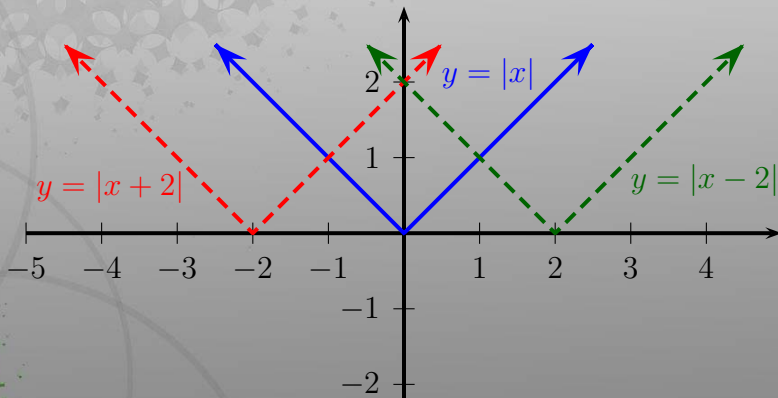
Stretching and Shrinking

Equation	$y = af(x)$ with $a > 1$	$y = af(x)$ with $0 < a < 1$
Effect on graph	The graph of $f(x)$ is obtained by **stretching** the graph of $y = f(x)$.	The graph of $f(x)$ is obtained by **shrinking** the graph of $y = f(x)$.

Consider the graphs of $f(x) = x^2$, $g(x) = 2x^2$ and $h(x) = \frac{1}{2}x^2$.
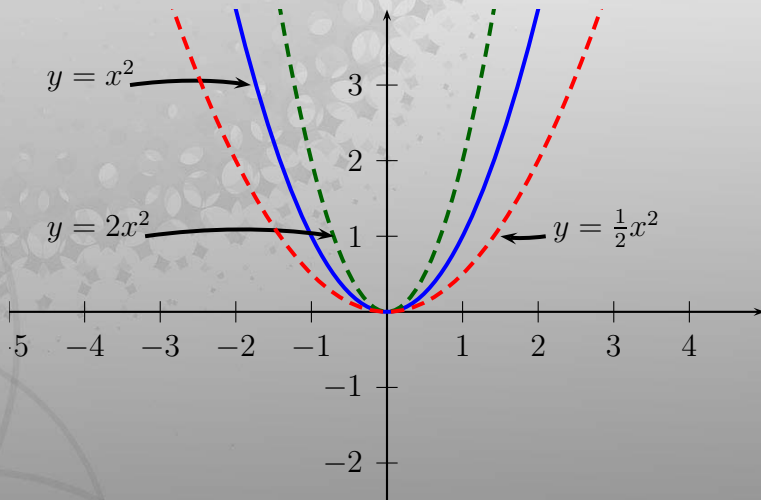
Multiple Transformations

When graphing a function containing more than one transformation perform the transformations in the following order:

- Left or right translation.

- Stretching or shrinking.

- Reflection in the x-axis.

- Upward or downward translation.

When you have to graph the function $f(x) = -2\sqrt{x-2}$.

1. **Start** with the graph of $y = \sqrt{x}$.
2. **Translate** it two units to the right to get the graph of $y = \sqrt{x-2}$.
3. **Stretch** this graph by a factor of two to get $y = 2\sqrt{x-2}$.
4. **Reflect** in the x-axis to get the graph $\boxed{y = -2\sqrt{x-2}}$.