Unit 4 Review -- Sequences and Series

Classifying Sequences

Classify each sequence as arithmetic, geometric, or neither. If arithmetic or geometric, find the common difference/ratio.

1. $-2, 6, -18, 54, \ldots$ Geom $r = -3$
2. $1, -1, -3, -5, \ldots$ Arith $d = -2$
3. $3, 6, -12, -24, \ldots$ neither
4. $9, 3, 1, \frac{1}{3}, \ldots$ Geom $r = \frac{1}{3}$

Recursive Definitions

Write a recursive definition for each sequence.

5. $8, 5, 2, -1, \ldots$ $a_1 = 8$, $a_n = a_{n-1} - 3$
6. $12, 4, \frac{4}{3}, \frac{4}{9}, \ldots$ $a_1 = 12$, $a_n = \frac{1}{3} a_{n-1}$
7. $-10, 20, -40, 80, \ldots$ $a_1 = -10$ $a_n = -2a_{n-1}$
8. $-1, 0, 1, 2, \ldots$ $a_1 = -1$ $a_n = a_{n-1} + 1$

Given the recursive definition, find the fourth term of each sequence.

9. $a_1 = 4$; $a_n = \frac{1}{9} a_{n-1}$ $4, \frac{4}{9}, \frac{4}{81}, \frac{4}{729}$
10. $a_1 = \frac{1}{5}$; $a_n = a_{n-1} + 3$ $\frac{1}{5}, 3\frac{1}{5}, 6\frac{1}{5}, 9\frac{1}{5}$ or $\frac{46}{5}$
11. $a_1 = -2$; $a_n = -3a_{n-1}$ $-2, 6, -18, 54$
12. $a_1 = 5$; $a_n = a_{n-1} - 17$ $5, -12, -29, -46$

Explicit Formulas

Write the explicit formula for each sequence.

13. $7, 3, -1, -5, \ldots$ $a_n = 7 + (n-1)(-4)$
14. $99, 33, 11, \frac{11}{3}, \ldots$ $a_n = 99\left(\frac{1}{3}\right)^{n-1}$
15. $-37, -29, -21, -13, \ldots$ $a_n = -37 + (n-1)(8)$
16. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \ldots$ $a_n = \frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$

Find the 10th term of each sequence.

17. $4, 8, 16, 32, \ldots$ $a_1 = 4$ $r = 2$ $a_{10} = 4(2)^9 = 2048$
18. $1, 4, 7, 10, \ldots$ $a_{10} = 1 + 3(9) = 28$
19. $\frac{1}{5}, -1, 5, -25, \ldots$ $a_{10} = \frac{1}{5}(-5)^9 = -390625$
20. $-8, -10, -12, -14, \ldots$ $a_{10} = -8 - 2(9) = -26$

Plan B, $58.83 more

Sums of Series

21. Find the sum of the series $40, 20, 10, 5, \ldots$ $S = \frac{40}{1 - \frac{1}{2}} = 80$
22. Find the sum of the first 10 terms of the series $18, 38, 58, 78, \ldots$ $a_{10} = 18 + 20(9) = 198$ $S_{10} = \frac{10}{2}(18 + 198) = 1080$
23. Find the sum of the first 8 terms of the series $3, -7, -17, -27, \ldots$ $a_8 = 3 - 10(7) = -67$ $S_8 = \frac{8}{2}(3 - 67) = -256$
24. Find the sum of the first 9 terms of the series $-1, 2, -4, 8, \ldots$ $S_9 = \frac{-1(1 - (-2)^9)}{1 - (-2)} = -171$
25. Find the sum of the series $\frac{1}{36}, \frac{1}{6}, 1, 6, \ldots$ diverges, no sum
26. Find the sum of the first 8 terms of the series $18, 12, 8, \frac{16}{3}, \ldots$ $S_8 = \frac{18\left(1 - \frac{2}{3}^8\right)}{1 - \frac{2}{3}} = \frac{12610}{243}$ or 51.893
27. Find the sum of the series $60, -15, \frac{15}{4}, -\frac{15}{16}, \ldots$ $S = \frac{60}{1 - (-\frac{1}{4})} = 48$

Applications of Sequences and Series

28. A theater has 30 seats in the first row of the center section. Each row behind the first row gains two additional seats. How many seats are in the 5th row in the center section?
$a_5 = 30 + 2(4) = 38$ seats

29. Mr. Carlson suffers from allergies. When allergy season arrives, his doctor recommends that he take 300 mg of his medication the first day, and decrease the dosage by half each day for one week. What is the amount of medication Mr. Carlson will take on the 7th day?
$a_7 = 300\left(\frac{1}{2}\right)^6 = \frac{75}{16}$ or 4.6875 mg

30. Lanie has decided to add strength training to her exercise program. Her trainer suggests that she add weight lifting for 5 minutes during her routine for the first week. Each week thereafter, she is to increase the weight lifting time by 2 minutes. How many minutes will she be devoting to weight lifting in week 10?
$a_{10} = 5 + 2(9) = 23$ min

31. There are 20 rows of seats on a concert hall: 25 seats are in the 1st row, 27 seats on the 2nd row, 29 seats on the 3rd row, and so on. If the price per ticket is \$2,300, how much will be the total sales for a one-night concert if all seats are taken?
$a_{20} = 25 + (2)(19) = 63$ $S_{20} = \frac{20}{2}(25 + 63) = 880$ \$2300(880) = \$2,024,000

32. A tube well is bored 800 meters deep. The 1st meter costs \$250 and the cost per meter increases by \$50 for every subsequent meter. Find the total cost incurred for the entire job.
$a_{800} = 250 + 50(799) = 40200$ $S_{800} = \frac{800}{2}(250 + 40200) = \$16{,}180{,}000$

33. You have won contest sponsored by a local radio station. If you are given the choice of the two payment plans. Plan A) \$1 on the first day, \$2 on the second day, \$3 on the third day, etc., for two weeks. Plan B) \$0.01 on the first day, \$0.02 on the second day, \$0.04 on the third day, etc. for two weeks. Which plan will pay you more? How much more?
$a_{14} = 1 + 1(13) = 14$ $S_{14} = \frac{14}{2}(1 + 14) = 105$ $S_{14} = \frac{.01(1 - 2^{14})}{1 - 2} = \$163.83$

34. Once a week Mrs. Baker makes sugar cookies. The first week she makes the recipe, she uses the full 2 cups of sugar called for. Each week after that, she reduces the amount of sugar by one third. How much sugar has she used for the cookies after five weeks.
$a_5 =$ $S_5 = \frac{2\left(1 - \frac{1}{3}^5\right)}{1 - \frac{1}{3}} = \frac{242}{81}$ or ~ 3 cups

Summation Notation

Evaluate.

35. $\sum_{x=1}^{5} (3x - 1)$
$= 2 + 5 + 8 + 11 + 14$
$= 40$

36. $\sum_{x=3}^{9} (6x^2)$
$= 54 + 96 + 150 + 216 + 294 + 384 + 486$
$= 1{,}680$