Precalculus Name Key

8.1 Notes: Sequences and Series- *Day 2*

If n is a positive integer, n FACTORIAL is defined as: $n! = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet 3 \bullet 2 \bullet 1$.

$n! = n(n-1)!$ $1! = 1(1-1)!$

As a special case, **zero factorial** is defined as: $0! = 1$.

if $n = 1 \longrightarrow$ $1 = 1(0)!$ $1 = 0!$

1. **Evaluate.** $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ $= 5040$	**2.** **Simplify the factorial expression.** $\frac{9!}{3!7!}$ $\frac{9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$ $\frac{72}{6} = 12$ OR: $\frac{9 \cdot 8 \cdot \cancel{7!}}{3 \cdot 2 \cdot 1 \cdot \cancel{7!}} = 12$
3. **Write the first 5 terms of the sequence.** $a_n = \frac{2^n}{n!}$ $a_1 = \frac{2^1}{1!} = \frac{2}{1} = 2$ $a_2 = \frac{2^2}{2!} = \frac{4}{2 \cdot 1} = 2$ $a_3 = \frac{2^3}{3!} = \frac{8}{3 \cdot 2 \cdot 1} = \frac{4}{3}$ $a_4 = \frac{2^4}{4!} = \frac{16}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{2}{3}$ $a_5 = \frac{2^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{4}{15}$ 2, 2, 4/3, 2/3, 4/15	**4.** **Simplify the factorial expression.** $\frac{(n+1)!}{n!}$ $\frac{(n+1)(\cancel{n})(\cancel{n-1})(\cancel{n-2})\ldots}{(\cancel{n})(\cancel{n-1})(\cancel{n-2})\ldots}$ $n+1$

A Series is the sum of the terms in a sequence. A series can be written with Summation notation where the sum of the first n terms of a sequence is represented by $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$, where i is called the index of summation, n is the upper limit and 1 is the lower limit.

Find the sum.

5. $\sum_{i=1}^{4}(4i+1)$ $4(1)+1 = 5$ $4(2)+1 = 9$ $4(3)+1 = 13$ $4(4)+1 = 17$ $5+9+13+17 = 44$ **How many terms are in this series?** 4	**6.** $\sum_{k=2}^{5}(2+k^3)$ $2+(2)^3 = 10$ $2+(3)^3 = 29$ $2+(4)^3 = 66$ $2+(5)^3 = 127$ $10+29+66+127 =$ 232 **How many terms are in this series?** 4	**7.** $\sum_{n=0}^{8}\left(\frac{1}{n!}\right)$ $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}$ $1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}+\frac{1}{5040}+\frac{1}{40320}$ $= 2.71828277 \approx e$ **How many terms are in this series?** 9
To find the number of terms in a series: (upper − lower) + 1		

A finite series is the sum of the first n terms of the sequence, which is also called a partial sum.

An infinite series is the sum of all the terms of the sequence.

Find the sum.

8. $\sum_{k=1}^{3}\left(\frac{3}{10^k}\right)$ $= \frac{3}{10^1} + \frac{3}{10^2} + \frac{3}{10^3}$ $= .3 + .03 + .003$ $= .333$ or $\frac{333}{1000}$ **third partial sum*	**9.** $\sum_{k=1}^{\infty}\left(\frac{3}{10^k}\right)$ $\frac{3}{10^1} + \frac{3}{10^2} + \frac{3}{10^3} + \frac{3}{10^4} + \frac{3}{10^5} + \ldots$ $.3 + .03 + .003 + .0003 + .00003 \ldots$ $= .33333\ldots$ $.\overline{3}$ or $\frac{1}{3}$
10. $\sum_{k=1}^{3} 5\left(\frac{1}{10^k}\right)$ $5\left(\frac{1}{10^1}\right) + 5\left(\frac{1}{10^2}\right) + 5\left(\frac{1}{10^3}\right)$ $5(.1) + 5(.01) + 5(.001)$ $.5 + .05 + .005$ $.555$ $\quad \frac{555}{1000} = \frac{111}{200}$ **third partial sum*	**11.** $\sum_{k=1}^{\infty} 5\left(\frac{1}{10^k}\right)$ $5\left(\frac{1}{10^1}\right) + 5\left(\frac{1}{10^2}\right) + 5\left(\frac{1}{10^3}\right) + 5\left(\frac{1}{10^4}\right) + \ldots$ $.5 + .05 + .005 + .0005 + \ldots$ $.55555\ldots$ $.\overline{5}$ or $5/9$