

Precalculus

Name Key

8.2 Notes: Arithmetic Sequences and Partial Sums

Arithmetic Sequences, also known as a discrete linear function, is a sequence for which consecutive terms have a **common difference**, d .

Determine whether or not the sequence is arithmetic. If it is, find the common difference.

1. $5, 8, 11, 14, 17, \dots$

$$\begin{array}{cccccc} \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ 3 & 3 & 3 & 3 & \end{array}$$

yes - arithmetic
 $d = 3$

2. $1, 4, 9, 16, 25, \dots$

$$\begin{array}{cccccc} \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ 3 & 5 & 7 & 9 & \end{array}$$

Not arithmetic

3. $1, \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \dots$

$$\begin{array}{cccccc} \frac{6}{6}, \frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}, \dots \\ \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & \end{array}$$

Yes arithmetic
 $d = \frac{1}{6}$

4. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

$$\begin{array}{cccccc} \frac{12}{12}, \frac{6}{12}, \frac{4}{12}, \frac{3}{12}, \dots \\ \checkmark & \checkmark & \checkmark & \checkmark \\ \frac{1}{12} & \frac{1}{6} & \frac{1}{3} & \frac{1}{4} & \end{array}$$

Not Arithmetic

Writing an explicit formula/rule for an arithmetic sequence a_n .

Fill in the missing terms from the sequence:

n	1	2	3	4	5	6	7	8
a_n	4	7	10	13	16	19	22	25

$\uparrow +3 \quad \uparrow +3 \quad \uparrow +3 \quad \uparrow +3$ $d=3$

Expanded:

$$a_2 = 4 + 3$$

$$a_3 = 4 + 3 + 3$$

$$a_4 = 4 + 3 + 3 + 3$$

* Repeat addition is multiplication!

Condensed:

$$a_2 = a_1 + 3(1)$$

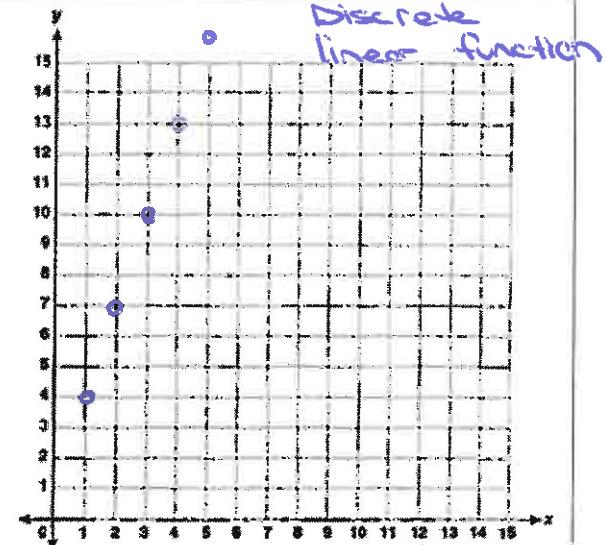
$$a_3 = a_1 + 3(2)$$

$$a_4 = a_1 + 3(3)$$

Arithmetic Explicit Rule:

$$a_n = a_1 + d(n-1)$$

\uparrow first term $\underbrace{\text{common difference}}$



* d relates to the constant rate of change \rightarrow slope

Write an explicit rule for the given sequence. Then answer any additional questions. Assume $n \geq 1$.

5. $5, 12, 19, 26, \dots$ $d = 7$

$$a_n = a_1 + d(n-1)$$

$$a_n = 5 + 7(n-1)$$

$$a_n = 5 + 7n - 7$$

$$a_n = 7n - 2$$

6. Find an explicit formula for a_n for the arithmetic sequence with the following terms:

$$a_3 = 19 \text{ and } a_5 = 27$$

* Not consecutive!

Find d using the constant rate of change: slope.

$$d = \frac{27-19}{5-3} = \frac{8}{2} = 4$$

$$a_n = a_1 + 4(n-1)$$

$$19 = a_1 + 4(3-1)$$

$$19 = a_1 + 8$$

$$11 = a_1$$

$$a_n = 4n + 7$$

$$a_n = 11 + 4(n-1)$$

7. 29, 25, 21, 17, 13, 9, ...

$$d = -4$$

$$a_n = 29 - 4(n-1)$$

$$a_n = 29 - 4n + 4$$

$$a_n = -4n + 33$$

9. Find the first five terms of the arithmetic sequence where $a_8 = 25$ and $a_{12} = 41$.

$$d = \frac{41-25}{12-8} = \frac{16}{4} = 4$$

$$a_n = a_1 + 4(n-1)$$

$$25 = a_1 + 4(8-1)$$

$$25 = a_1 + 28 \quad a_1 = -3$$

$$a_n = -3 + 4(n-1)$$

$$-3, 1, 5, 9, 13$$

Arithmetic Series

Find the sum of: $40+37+34+31+28+25+22$

The sum of a finite arithmetic sequence with n terms (n^{th} partial sum) can be found by:

$$S_n = \frac{n}{2}(a_1 + a_n) \quad \text{where } n = \text{number of terms} \quad a_1 = \text{first term} \quad a_n = \text{last term}$$

* not necessarily the first position

Find the sum of the finite arithmetic sequence.

11. Sum of integers from 1 to 35.

$$1+2+3+4+\dots+35$$

Arithmetic b/c $d=1$

35 terms

$$a_1 = 1$$

$$a_{35} = 35$$

$$S_{35} = \frac{35}{2}(1+35)$$

$$S_{35} = 630$$

13. 50th partial sum of the arithmetic sequence

$$-6, -2, 2, 6, \dots$$

$$d=4$$

$$S_{50} = \frac{50}{2}(-6 + a_{50})$$

missing

$$S_{50} = \frac{50}{2}(-6 + 190)$$

$$S_{50} = 4600$$

12. Sum of odd integers from 1 to 57

$$1+3+5+\dots+57 \quad d=2$$

How many terms?

$$a_n = a_1 + d(n-1)$$

$$57 = 1 + 2(n-1)$$

$$56 = 2(n-1)$$

$$28 = n-1$$

$$29 = n$$

$$S_{29} = \frac{29}{2}(1+57)$$

$$S_{29} = 841$$

14. Determine the seating capacity of an auditorium with 30 rows of seats if there are 20 seats in the first row, 22 in the second, 24 in the third row, and so on.

$$d=2$$

$$a_{30} = 20 + 2(30-1)$$

$$a_{30} = 78$$

$$S_{30} = \frac{30}{2}(20+78)$$

$$S_{30} = 1470 \text{ Seats}$$

15. $\sum_{n=1}^{100} (2+3n)$

Arithmetic rule

$$a_1 = 2+3(1) = 5$$

$$a_{100} = 2+3(100) = 302$$

100 terms in the series.

$$S_{100} = \frac{100}{2}(5+302)$$

$$S_{100} = 15350$$

8. 11, 5, -1, -7, -13, -19, ...

$$d = -6$$

$$a_n = 11 - 6(n-1)$$

$$a_n = 11 - 6n + 6$$

$$a_n = -6n + 17$$

10. Find the 10th term of the arithmetic sequence whose first two terms are 8 and 20.

$$d=12$$

$$a_n = 8 + 12(n-1)$$

$$a_{10} = 8 + 12(10-1)$$

$$a_{10} = 8 + 12(9)$$

$$a_{10} = 116$$

$$S_7 = \frac{7}{2}(40+22) = 217$$

$$20+22+24+26+\dots+a_{30}$$

16.

$$\sum_{n=1}^{100} (2+3n)$$

Arithmetic rule

$$a_{21} = 2+3(21) = 65$$

$$a_{100} = 2+3(100) = 302$$

80 terms!

$$S_{100} = \frac{80}{2}(65+302)$$

$$S_{100} = 14680$$