

8.3 Notes: Geometric Sequences and Series

A sequence is _____ if the ratios of consecutive terms are the same. So

$a_1, a_2, a_3, \dots, a_n$ is geometric if there is a number r such that $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots = r : r \neq 0$.

Determine whether each sequence is geometric. If so, find r .

1. 3, -9, 27, -81, 243...	2. $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \dots$	3. 1, 2, 6, 24, 120...
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Fill in the blanks with the missing terms of the geometric sequence.

4. 144, _____, 36, _____, 9, 4.5		
5. 5, 15, 45, _____, _____		
Part 1: $a_2 = 5 \cdot$ $a_3 = 5 \cdot$ $a_4 = 5 \cdot$	Part 2: $a_2 = a_1 \cdot$ $a_3 = a_1 \cdot$ $a_4 = a_1 \cdot$ $a_n =$	Part 3: Find a_{10}

In a geometric sequence, we repeatedly _____ to get successive terms. Hence, to write an explicit formula for the n^{th} term, we must have a power of r .

$$a_n =$$

Examples

6. Find the n^{th} term of the geometric sequence. Write the first five terms of the geometric sequence whose first term is 2 and whose common ratio is 4.	7. Find the 9 th term of the geometric sequence whose first term is 4 and whose common ratio is $\frac{1}{2}$.	8. Find the n^{th} term of the geometric sequence and then find the value of the tenth term. 6, -2, $\frac{2}{3}, \dots$
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9. Find the value of the sixth term of a sequence whose second term is -18 and the fifth term is $\frac{2}{3}$.

A _____ is the sum of the terms of a geometric sequence.

Sum of a *finite* geometric sequence

$$S_n = \sum_{i=1}^n a_1 r^{i-1} = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$r = \underline{\hspace{4cm}}$$

$$n = \underline{\hspace{4cm}}$$

$$a_1 = \underline{\hspace{4cm}}$$

Sum of an *infinite* geometric sequence

****If $|r| < 1$, then $S_\infty = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1-r}$**

****If $|r| > 1$, the series does not have a sum.**

Determine whether each series is infinite or finite. Then, find each sum.

10. $\sum_{n=1}^7 2^{n-1}$

11. $\sum_{n=1}^{\infty} 4(0.6)^{n-1}$

12. $\frac{3}{4} + \frac{15}{8} + \frac{75}{16} + \dots$

13. $\frac{2}{3} + \frac{1}{3} + \frac{1}{6}$

14. $4 + 2 + 1 + \frac{1}{2} + \dots + \frac{1}{32}$

15. $5 + .5 + .05 + .005 \dots$