Pre-Calculus
Unit : Parametric and Polar Equations (7)
Text References: Pre-Calculus with Limits; Larson, Hostetler, Edwards.

By the end of the unit, you should be able to complete the problems below. The teacher may provide additional material during the unit. While not every assignment will be collected or graded, you will still be responsible for knowing how to do, explain, and evaluate every problem listed.

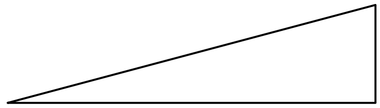
For the duration of the study of this unit, you will need to keep all of the work that you accumulate while learning. This includes homework, quizzes, tests, any additional worksheets.

Text Section	Objectives	Assignment
	Trig Review	w/s
9.5	❐Be able to sketch a function defined parametrically. (by hand and by calculator) ❐Be able to find the rectangular representation of a function defined parametrically.	p. 673 (1-30)
9.6	❐Plot points in the polar coordinate system	p.680 (1-20)
9.6	❐Convert between polar and rectangular coordinates. ❐Convert between rectangular and polar equations.	p.680 (21-32,39 – 70)
	Review	w/s

Pre-Calculus **Name: ______________________________**
Overview – Trigonometry Review

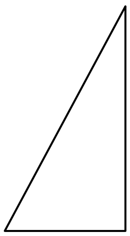
Trig Review:

I. SOH CAH TOA

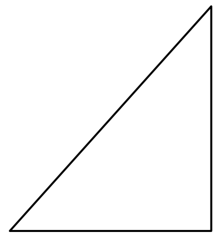


II. Special Right Triangles

a. 30 – 60 – 90



b. 45 – 45 – 90



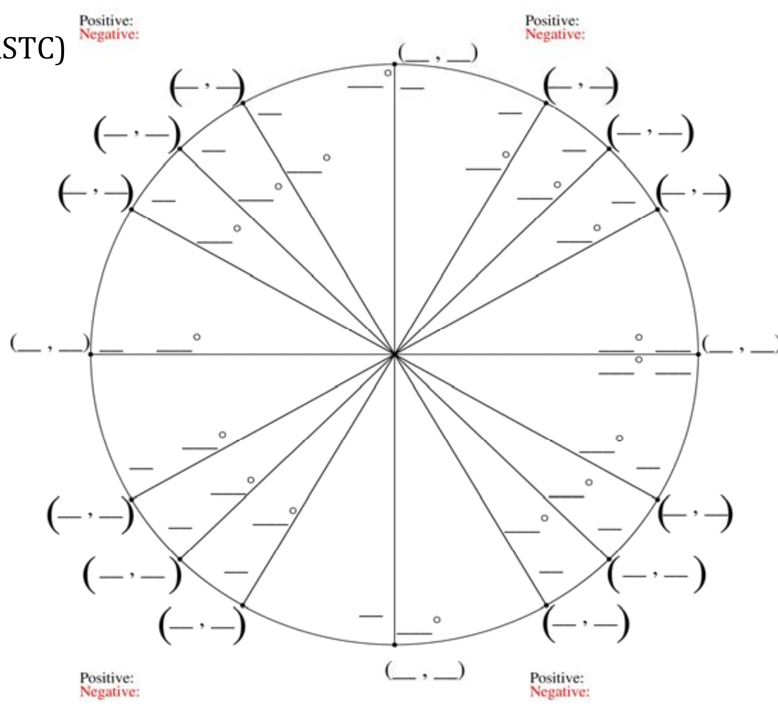
III. Examples:

a. Find $\sin 210^\circ$

b. Find $\cos 300^\circ$

c. Find $\tan 120^\circ$

IV. Radians (Recall: $180 = \pi$ and ASTC)



EmbeddedMath.com

V. Examples:

a. Find $\cos\frac{7\pi}{6}$

b. Find $\cos\frac{3\pi}{2}$

c. Find $\sin\frac{5\pi}{4}$

d. Find $\sin\left(-\frac{\pi}{4}\right)$

e. Given $y = 6\cos\theta \; and \; x = 2\sin\theta$, find x and y if:

i. $\theta = \pi$

ii. $\theta = \frac{5\pi}{6}$

iii. $\theta = -\frac{5\pi}{3}$

iv. $\theta = -\frac{3\pi}{2}$

v. $\theta = \frac{11\pi}{6}$

vi. $\theta = \frac{3\pi}{4}$

Trigonometry
Trig values of Radians - Additional Practice

Find the value of each trig function. Draw the angle on a coordinate system if necessary.

1. $\sin\frac{5\pi}{4}$

2. $\cos\frac{2\pi}{3}$

3. $\sin\frac{\pi}{4}$

4. $\cos\frac{\pi}{6}$

5. $\sin\left(-\frac{5\pi}{3}\right)$

6. $\cos(-\pi)$

7. $\tan\left(-\frac{7\pi}{4}\right)$

8. $\tan\frac{4\pi}{3}$

9. $\cos\frac{7\pi}{6}$

10. $\cos\frac{3\pi}{4}$

11. $\tan 2\pi$

12. $\sin\pi$

13. $\tan\left(-\frac{11\pi}{6}\right)$

14. $\sin\frac{3\pi}{2}$

15. $\cos\frac{5\pi}{6}$

16. $\sin\frac{11\pi}{6}$

17. $\cos\frac{\pi}{3}$

18. $\tan\left(-\frac{\pi}{2}\right)$

Homework: Worksheet

Pre-Calculus **Name:** ______________________________

9.5 - Parametric Equations

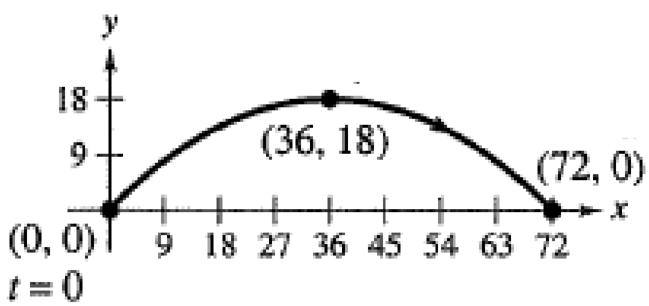
Parametric curves are relations $(x(t), y(t))$ for which both *x* and *y* are defined as functions of a <u>**third**</u> variable, *t*. As in $x = f(t)$ and $y = g(t)$. Essentially, we now will have the ability to not only tell <u>where</u> an object is given a point (*x*, y), but also <u>when</u> the object is at that point (*x*, *y*).

Given:

$$y = -\frac{x^2}{72} + x$$

With the Parameter:

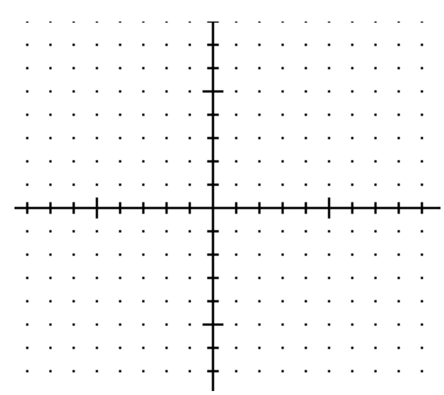
$$x = 24\sqrt{2}t$$
$$y = -16t^2 + 24\sqrt{2}t.$$



Sketch by hand. Indicate direction. Find the domain and range....

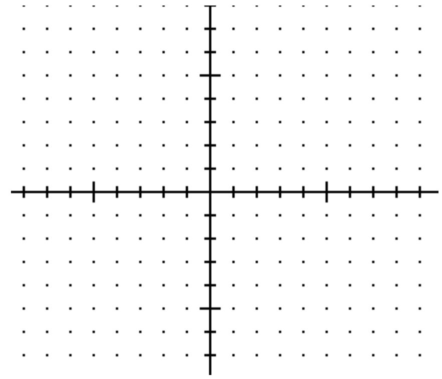
Example 1: Sketch the curve given by the parametric equations: $\begin{cases} x = t^2 - 4 \\ y = \dfrac{t}{2} \end{cases}$, for $-2 \le t \le 3$

t						
x						
y						



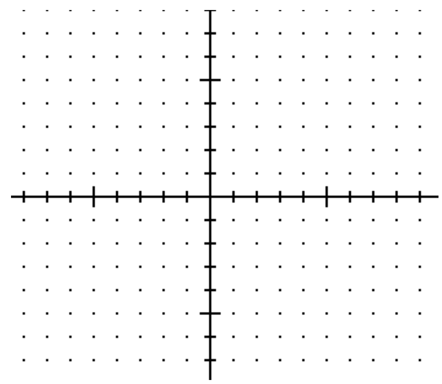
Example 2: A particle moving in the coordinate plane in such a way that $x(t)=3t-6$ and $y(t)=t-4$ for $0\le t\le 5$.

t						
x						
y						



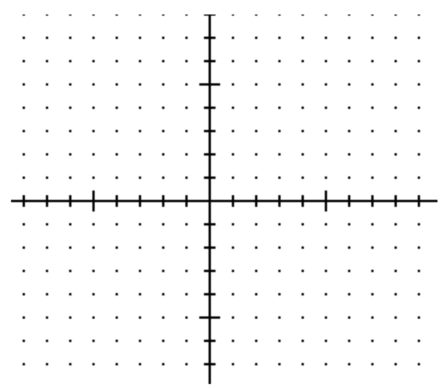
Example 3: Sketch the curve and the orientation given by $x(t)=2-t$ and $y(t)=t^2$ for $-2\le t\le 3$

t						
x						
y						



Example 4: Sketch the curve and the orientation given by $x(t)=2\cos\theta$ and $y(t)=6\sin\theta$ for $\theta=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\pi,\frac{5\pi}{4},\frac{3\pi}{2},2\pi$.

T								
x								
y								



Use a graphing calucator….

Example 5 : Use a graphing calucator to graph the curves represented by the parametric equations. Tell which curves are functions.

a. $\begin{cases} x=t^2 \\ y=t^3 \end{cases}$

b. $\begin{cases} x=t \\ y=t^3 \end{cases}$

c. $\begin{cases} x=t^2 \\ y=t \end{cases}$

d. $\begin{cases} x=t^2 \\ y=2+t \end{cases}$, from $-3\le t\le 3$

e. $\begin{cases} x=4\sin\theta \\ y=3\cos\theta \end{cases}$

Eliminate the parameter….

Example 6: Find the Cartesian (rectangular) equations of the curve. Identify the curve represented by the equations.

a. $\begin{cases} x = \dfrac{1}{\sqrt{t+1}} \\ y = \dfrac{t}{t+1} \end{cases}$

b. $\begin{cases} x = 3\cos\theta \\ y = 4\sin\theta \end{cases}$, for $0 \le \theta \le 2\pi$

c. $x = t - 2$ and $y = \dfrac{1}{t-1}$

d. $x = t^2 - 1$ and $y = t + 2$

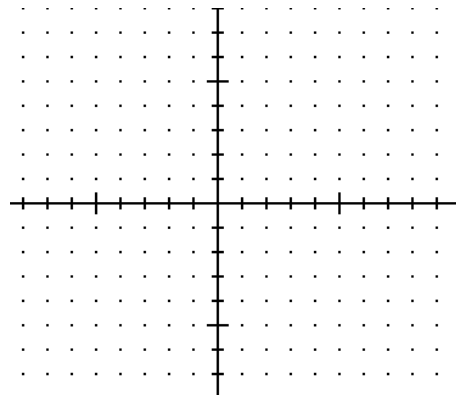
e. $x = t^2 + 2$ and $y = t^2 - 1$

f. $x = 3 + 2\cos\theta$ and $y = 1 + \sin\theta$

9.5 - Parametric Equations – Additional Practice

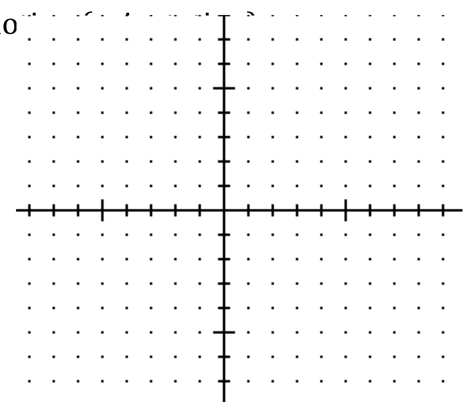
1) A particle moving in the coordinate plane in such a way that $x = t - 2$ and $y = 6 - 2t$ for $0 \le t \le 5$. Sketch the path of the particle and indicate the direction of motion (orientation)

t						
x						
y						



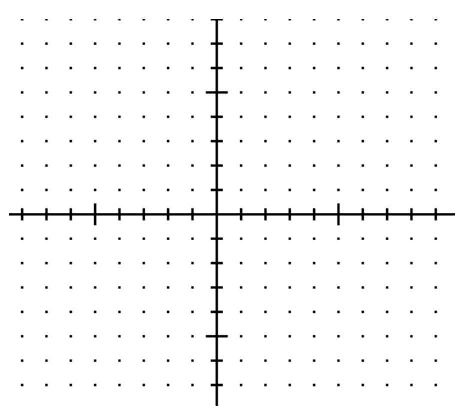
2) A particle moving in the coordinate plane in such a way that $x = t^2$ and $y = t - 2$ for $-2 \le t \le 3$. Sketch the path of the particle and indicate the direction of mo

t						
x						
y						



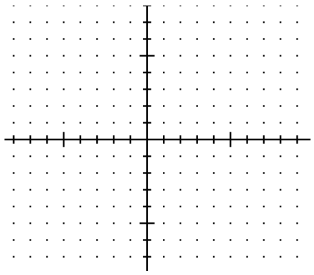
3) A particle moving in the coordinate plane in such a way that $x = 4\cos^2\theta$ and $y = 2\sin\theta$ for $\theta = -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}$. Sketch the path of the particle and indicate the direction of motion (orientation)

θ						
x						
y						

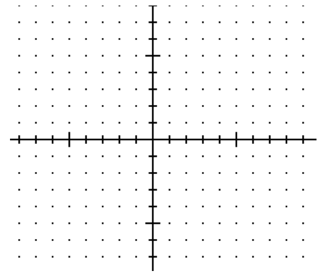


4) Use your calculator to graph the parametric equations

a. $x=\sqrt{t},\ y=1-t$



b. $x=4+2\cos\theta,\ y=-1+2\sin\theta$



5) A parametric curve is defined is defined by $x=3-2t,\ y=2+3t$. Find the rectangular equation of the curve. Solve the equation for *y*.

6) A parametric curve is defined is defined by $x=\cos\theta,\ y=3\sin\theta$. Find the rectangular equation of the curve. What conic section does this represent?

7) A parametric curve is defined is defined by $x=3\cos\theta,\ y=3\sin\theta$. Find the rectangular equation of the curve. What conic section does this represent?

8) A parametric curve is defined is defined by $x=\ln 2t,\ y=2t^2$. Find the rectangular equation of the curve.

9) A segment (*x, y*) is determined by the parametric equations for $x = 6t,\ y = 2t + 1$ for $-4 \le t \le 5$
where *t* represents real numbers.

a. Find the restrictions on *x* (Domain)

b. Find the restrictions on *y* (Range)

c. Find the rectangular equation of the curve.

10) A segment (*x, y*) is determined by the parametric equations for $x = 3t - 3,\ y = 4t - 5$ for
$-2 \le t \le 3$
where *t* represents real numbers.

a. Find the restrictions on *x* (Domain)

b. Find the restrictions on *y* (Range)

c. Find the rectangular equation of the curve.

11) A segment (*x, y*) is determined by the parametric equations for $x = 5 - 3t,\ y = \frac{1}{2}t - 4$ for
$-1 \le t \le 4$
where *t* represents real numbers.

a. Find the restrictions on *x* (Domain)

b. Find the restrictions on *y* (Range)

c. Find the rectangular equation of the curve.

12) A segment (x, y) is determined by the parametric equations for $x = -7t,\ y = 5t - 1$ for $1 \le t \le 6$ where t represents real numbers.

a. Find the restrictions on x (Domain)

b. Find the restrictions on y (Range)

c. Find the rectangular equation of the curve.

d. What is the slope of the line found in "c"?

e. What is the y – intercept of the line found in "c"?

14) A segment (x, y) is determined by the parametric equations for $x = \frac{1}{3}t - 1,\ y = 6t - 2$ for $-3 \le t \le 3$
where t represents real numbers.

a. Find the restrictions on x (Domain)

b. Find the restrictions on y (Range)

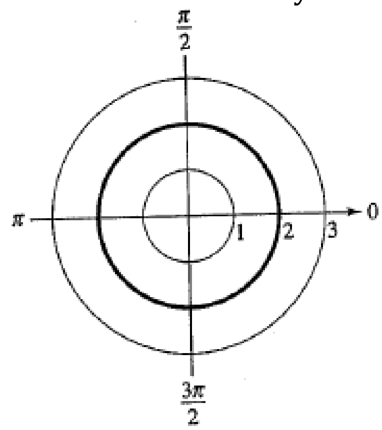
c. Find the rectangular equation of the curve.

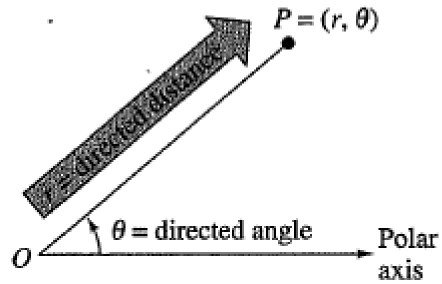
d. What is the slope of the line found in "c"?

e. What is the y – intercept of the line found in "c"?

Pre-Calculus **Name: ______________________________**
9.6 - Polar Coordinates

I. Polar coordinate system

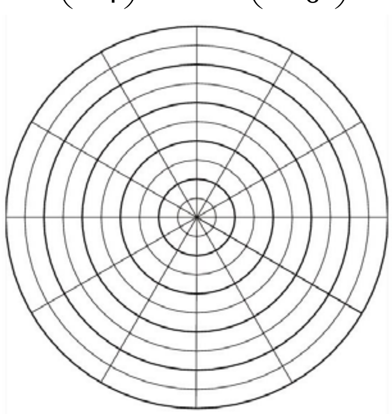


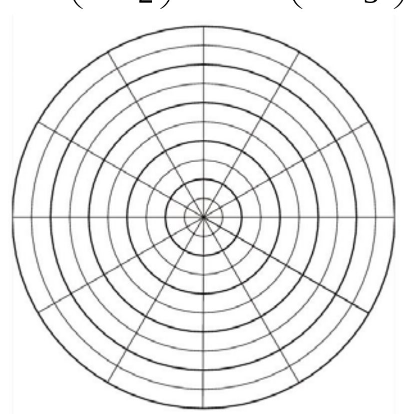


Plotting Points in the Polar Coordinate System

Example 1: Plot A and B on the first graph, then C and D on the second graph.

$A = \left(1, \frac{\pi}{4}\right)$ $B = \left(5, \frac{7\pi}{6}\right)$ $C = \left(3, -\frac{\pi}{2}\right)$ $D = \left(-4, \frac{2\pi}{3}\right)$

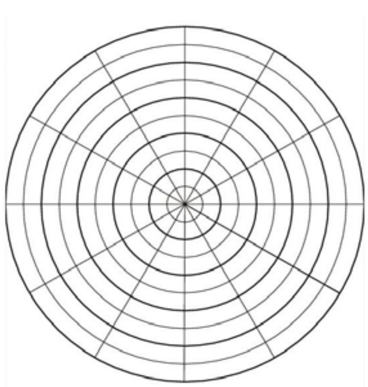




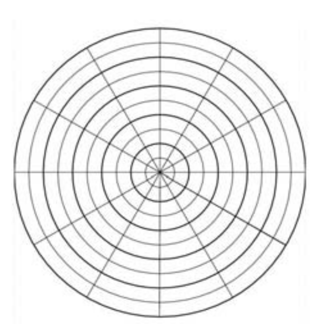
Unlike the Cartesian coordinate system, in which each point on the plane is expressed by a unique coordinate pair, each point in the coordinate plane can be represented by an infinite number of polar coordinate pairs.

Example 2.

A. Find and graph two polar coordinate pairs that coordinate pairs that represents the same point on the plane as $\left(2, \frac{\pi}{6}\right)$.



B. Find and graph polar represent the same point on the plane as $\left(-3, -\frac{2\pi}{3}\right)$ on the interval $[0, 2\pi]$.

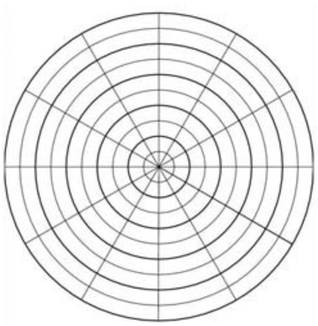


Coordinate Conversion

Recall: $x = r\cos\theta$ $y = r\sin\theta$ $r = \sqrt{x^2 + y^2}$ $\tan\theta = \frac{y}{x}$

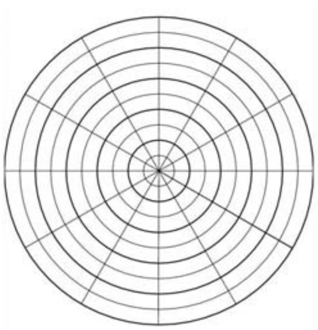
Example 3: Convert each point to rectangular coordinates. Graph the polar coordinates.

a. $(2, \pi)$ b. $\left(\sqrt{3}, \frac{\pi}{6}\right)$ c. $(-3, \pi)$ d. $\left(7, \frac{4\pi}{3}\right)$ e. $\left(-2, \frac{7\pi}{4}\right)$



Example 4: Rectangular-to-Polar Conversion. Graph the polar coordinates.

a. $(-1, 1)$ b. $(0, 2)$ c. $(4, -4)$ d. $(-1, \sqrt{3})$

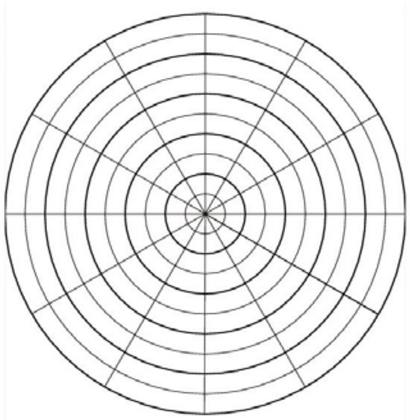


Equation Conversion

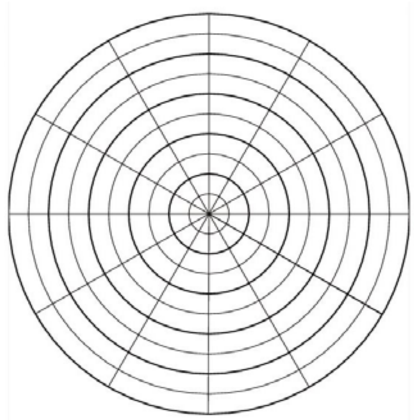
Example 5:
Describe the graph of each polar equation and find the corresponding rectangular equation.

a. r = 2 b. $\sin\theta = \frac{\pi}{3}$ c. $r = \sec\theta$

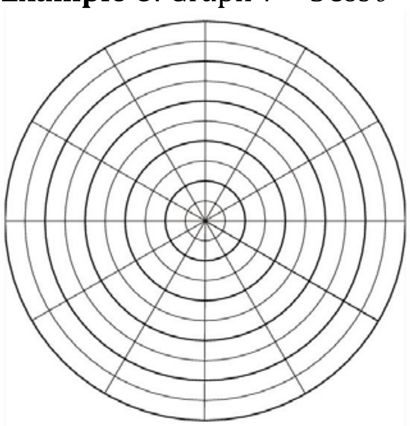
Example 6: $r = a\cos\theta$



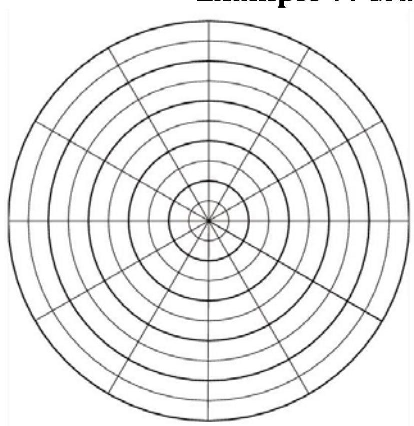
Example 7: $r = a\sin\theta$



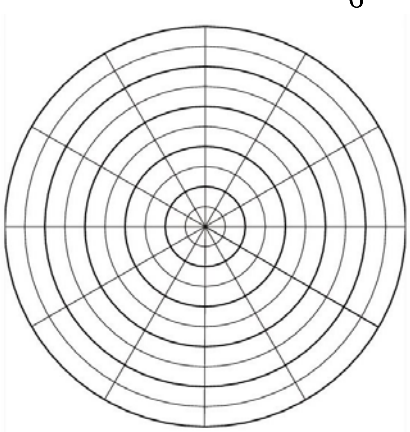
Example 6: Graph $r = 3\cos\theta$



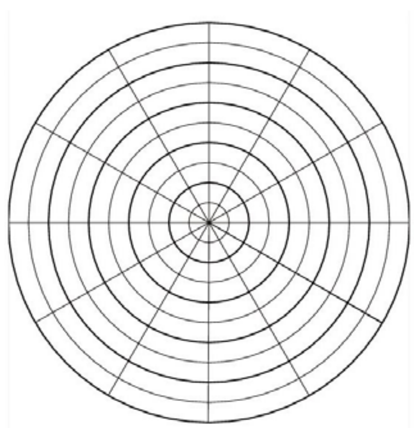
Example 7: Graph $r = 5$



Example 6: Graph $\theta = -\frac{\pi}{6}$



Example 7: Graph $r = 4\cos 2\theta$

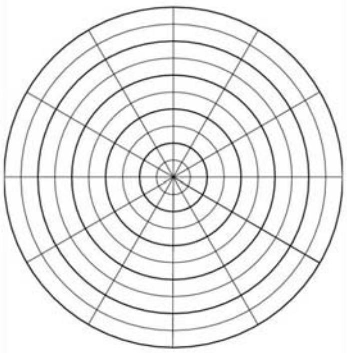


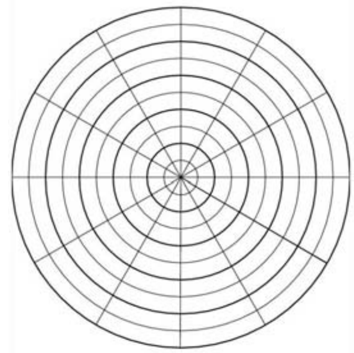
9.6 - Polar Coordinates – Additional Practice

Remember: $x = r\cos\theta$ $y = r\sin\theta$ $r = \sqrt{x^2 + y^2}$ $\tan\theta = \frac{y}{x}$

1) Plot A, B, and C on the first graph, then D, E, and F on the second graph.

$A = \left(2, \frac{5\pi}{4}\right)$ $B = (3, 3\pi)$ $C = \left(5, -\frac{2\pi}{3}\right)$ $D = \left(-3, -\frac{3\pi}{4}\right)$ $E = \left(-2, \frac{11\pi}{6}\right)$ $F = \left(3, \frac{4\pi}{3}\right)$





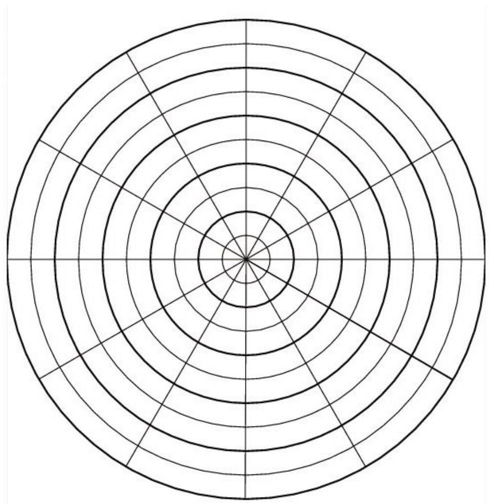
2) Find two polar coordinate pairs that represent the same point on the plane as $\left(3, \frac{5\pi}{4}\right)$. Then graph.

3) Find a polar coordinate pair that represent the same point on the plane as $\left(-2, \frac{3\pi}{2}\right)$ on $[0, 2\pi]$. Then graph.

5) Find a coordinate pair that represents the same point on the plane as $\left(-4, -\frac{\pi}{6}\right)$ on $[0, 2\pi]$. Then graph.

6) Find a coordinate pair that represents the same point on the plane as $\left(5, \frac{7\pi}{4}\right)$ on $[0, 2\pi]$. Then graph.

7) Find a coordinate pair that represents the same point on the plane as $\left(1, -\frac{\pi}{3}\right)$ on $[0, 2\pi]$. Then graph.



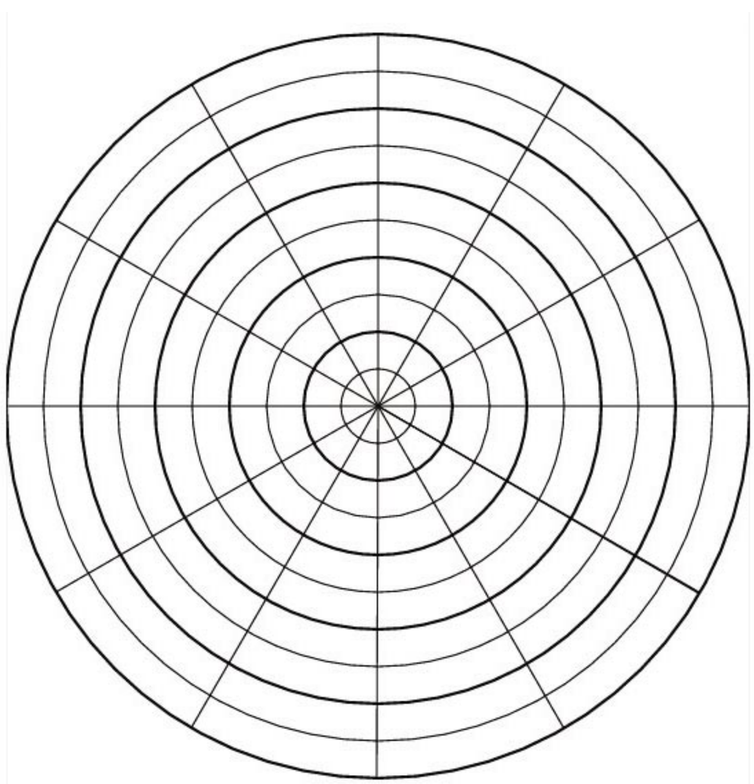
9) Convert the point $\left(2, \frac{\pi}{3}\right)$ from polar to rectangular coordinates. Then graph.

10) Convert the point $\left(-\sqrt{2}, \frac{5\pi}{4}\right)$ from polar to rectangular coordinates. Then graph.

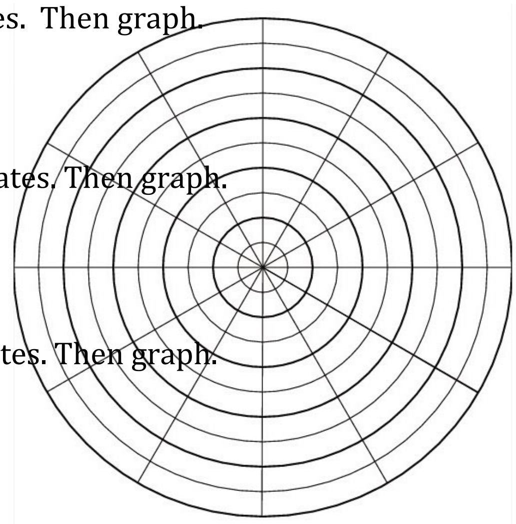
11) Convert the point $\left(-2, \frac{3\pi}{4}\right)$ from polar to rectangular coordinates. Then graph.

12) Convert the point $(1, \pi)$ from polar to rectangular coordinates. Then graph.

13) Convert the point $\left(3, \frac{5\pi}{3}\right)$ from polar to rectangular coordinates. Then graph.



15) Convert the point (2, -2) from rectangular to polar coordinates. Then graph.

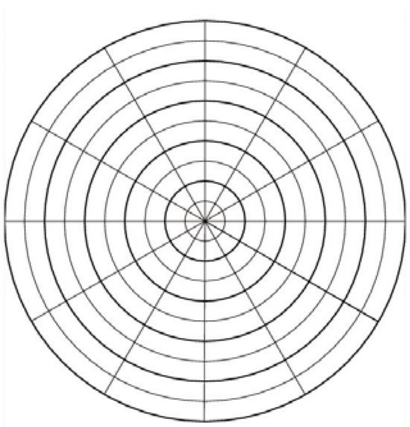


16) Convert the point $\left(-1, \sqrt{3}\right)$ from rectangular to polar coordinates. Then graph.

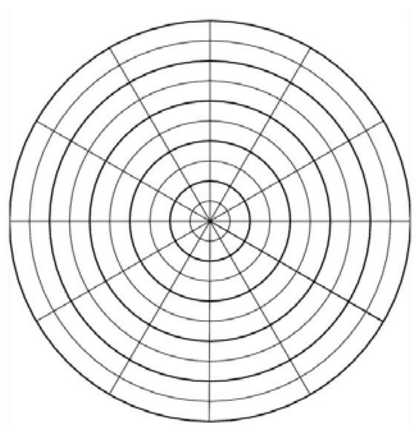
17) Convert the point $\left(3\sqrt{3}, 3\right)$ from rectangular to polar coordinates. Then graph.

18) Convert the point (-1, 0) from rectangular to polar coordinates. Then graph.

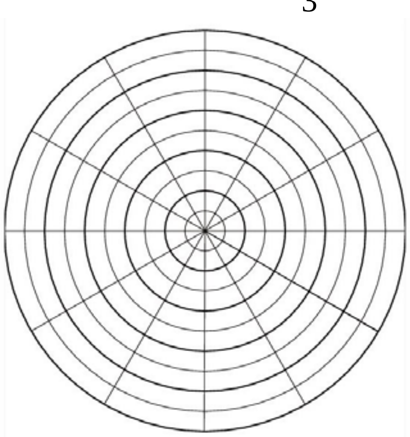
20) Create a rough graph of the polar curve $r = 4\cos\theta$



21) Create a rough graph of the polar curve $r = 2$



22) Create a rough graph of the polar curve $\theta = \frac{4\pi}{3}$



23) Create a rough graph of the polar curve $r = 1 + \sin\theta$