Precalculus Notes- Introduction to Limits **Name ____________________________**

<u>Definition of Limit</u> [c, L are real numbers]: as *x* <u>approaches</u> *c*, the limit of *f(x)* equals *L* $\lim_{x \to c} f(x) = L$

Method 1: Graphing	Examples
Approaching from BOTH the left and right side of x = c, follow the graph of the function towards the value of c. *What does the y-value "approach" as the function approaches the given x-value?* Any value of the function at *x = c* does not affect whether there is a limit.	
Method 2: Table of Values	**Examples**
What does the y-value "approach" as the function approaches the given x-value? Any value of the function at *x = c* does not affect whether there is a limit. *x* | *y*	
One-Sided Limits	**Examples**
<u>One-Sided Limits:</u> [c, L are real numbers] as *x approaches c* from left OR right, the limit of *f(x)* equals *L* <u>Left-handed limit</u>: $\lim_{x \to c^-} f(x) = L$ <u>Right-handed limit</u>: $\lim_{x \to c^+} f(x) = L$ ____________________ $\lim_{x \to c} f(x) = L$ **if and only if** $\begin{cases} \lim_{x \to c^+} f(x) = L \\ \text{AND} \\ \lim_{x \to c^-} f(x) = L \end{cases}$ **BOTH right and left-hand limits are equal*	

Limits at Vertical Asymptotes	Examples
<u>Limits at VA:</u> as *x approaches c* from left OR right, the limit of *f(x)* equals ∞ or -∞ <u>Limit from left</u>: $\lim_{x\to c^-} f(x)=\infty \text{ or } -\infty$ <u>Limit from right</u>: $\lim_{x\to c^+} f(x)=\infty \text{ or } -\infty$	

Limits at Infinity	Examples
<u>Limits at Infinity:</u> as *x* approaches ∞ or -∞, the limit of *f(x)* equals *L* $\lim_{x\to\infty} f(x)=L$ **OR** $\lim_{x\to-\infty} f(x)=L$ Limits of a function at infinity can approach a number, ∞ or -∞	

Limits that fail to exist (DNE)	Examples
1. *f(x)* approaches different limits from left and right 2. function oscillates between 2 numbers an infinite number of times	

Properties of Limits Given: Limits L and M are real numbers, c and k are real numbers $\lim_{x\to c} f(x)=L$ and $\lim_{x\to c} g(x)=M$

1. <u>Sum Rule</u>: $\lim_{x\to c}[f(x)+g(x)]=\lim_{x\to c} f(x)+\lim_{x\to c} g(x)=L+M$

2. <u>Difference Rule</u>: $\lim_{x\to c}[f(x)-g(x)]=\lim_{x\to c} f(x)-\lim_{x\to c} g(x)=L-M$

3. <u>Product Rule</u>: $\lim_{x\to c}[f(x)\bullet g(x)]=\lim_{x\to c} f(x)\bullet\lim_{x\to c} g(x)=L\bullet M$

4. <u>Constant Multiple Rule</u>: $\lim_{x\to c} k\bullet f(x)=k\bullet\lim_{x\to c} f(x)=k\bullet L$

5. <u>Quotient Rule</u>: $\lim_{x\to c}\frac{f(x)}{g(x)}=\frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}=\frac{L}{M}, M\neq 0$

6. <u>Power Rule</u>: $\lim_{x\to c}[f(x)]^{r/s}=\left[\lim_{x\to c} f(x)\right]^{r/s}=L^{r/s}$, if r/s is real, r and s are integers, $s\neq 0$

Algebraic Method 1: Direct Substitution	**Examples**
$\lim_{x \to c} f(x) = f(c)$ -polynomial functions (linear, quad, cubic, etc) -easy radical functions -exponential functions -rational functions when denominator ≠ 0 -when c is in the domain	
Algebraic Method 2: Rational Functions	**Examples**
-Use when you have a rational function -use when you substitute and you get $\frac{0}{0}$ -simplify the fraction *dividing out method OR LCD -substitute into simplified fraction	
Algebraic Method 3: Rationalize the Numerator	**Examples**
-Use when you have a radical function -use when you substitute and denominator = 0 -multiply top and bottom by conjugate of top -simplify (original denominator will reduce out) -substitute into simplified fraction	

Special Methods		
Right and Left- Hand –instead of graphing - use for limits as $x \to c$, not at infinity -use for point of discontinuity or VA -piecewise function, absolute value in fraction 1. Approaching from left and right, substitute numbers closer and closer to *c* 2. Limit is ∞, -∞, or DNE	End Behavior Model - without graphing -use for limits c as $x \to \infty$ or $-\infty$ -use for rational functions form *p(x)/q(x)* -find the horizontal or slant asymptote 1. Write function as a single fraction 2. Long divide to find asymptote 3. The end behavior of the asymptote models the end behavior of the function HA: Limit is a number or 0 Slant: Limit is ∞ or -∞	"Significant Parts" (made up this name) -without graphing -use for limits at infinity as $x \to \infty$ or $-\infty$ -use for functions with separate "parts" 1. Check each part of the function. Does it increase, decrease, or approach 0 as x approaches ∞ or -∞? 2. Which part is significant and affects the limit?