Common Core Math 3
Trigonometry

Thomas Paine
1737-1809

"The scientific principles that man employs to obtain the foreknowledge of an eclipse, or of any thing else relating to the motion of the heavenly bodies, are contained chiefly in that part of science that is called ***trigonometry****, or the properties of a triangle, which, when applied to the study of the heavenly bodies, is called astronomy; when applied to direct the course of a ship on the ocean, it is called navigation; when applied to the construction of figures drawn by a ruler and compass, it is called geometry; when applied to the construction of plans of edifices, it is called architecture; when applied to the measurement of any portion of the surface of the earth, it is called land-surveying. In fine, it is the soul of science. It is an eternal truth: it contains the* mathematical demonstration *of which man speaks, and the extent of its uses are unknown."*

Name: ______________________________________

WAKE COUNTY
PUBLIC SCHOOL SYSTEM

APEX HIGH SCHOOL
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Common Core Math 3
Unit 6 Trigonometry

Day	Date	Homework
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Common Core Math 3 – Unit 6 Trigonometry

Topics in this unit:

- **Circles**
  - Equation of a circle
  - Length of an arc
  - Area of a sector
  - Central, inscribed, and outside angles
- **Angles and the Unit Circle**
  - Angle measures in degrees/minutes/seconds and radians
  - Positive and negative angles
  - Coterminal angles
  - Reference angle
- **Trigonometric functions**
  - Sine, cosine, tangent and the unit circle
  - Cosecant, secant, cotangent (H)
  - Graphing sine, cosine
  - Graphing tangent (H)
  - Model real-world problems

Students will be able to . . .

- Graph sine and cosine and tangent functions by hand in simple cases and using technology for more complicated cases, showing period, midline, and amplitude, vertical shift.
- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers.
- Convert radians to degrees and degrees to radians.
- Convert decimals degrees to degrees, minutes, and seconds. (H)
- Find coterminal angles in degrees and radians, positive and negative.
- Create trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and derive the other two Pythagorean identities. (H)
- Know the reciprocal identities (Cosecant, Secant, Cotangent). (H)
- Derive the equation of a circle of given center and radius using the Pythagorean Theorem.
- Complete the square to find the center and radius of a circle given by an equation.
- Find the length of a circular arc, and the area of a sector.
- Identify and describe relationships among inscribed angles, radii, and chords.
- Prove properties of angles for a quadrilateral inscribed in a circle (Opposite angles of an inscribed quadrilateral are supplementary.)

Vocabulary

A **circle** is the set of all points equidistant from a given point called the center.

A **unit circle** has its center at the origin and a radius of 1.

The **initial side** of an angle is the ray that remains fixed; the angle of rotation begins from this ray.

The **terminal side** of an angle is the ray that rotates away from the initial side.

A **positive angle** has counterclockwise rotation.

A **negative angle** has a clockwise rotation.

The angle measure of a full rotation is 360 **degrees** or 2π **radians**.

An angle in **standard position** has its vertex at the origin and its initial side along the positive x-axis.

A **quadrantal angle** is an angle in standard position whose terminal side coincides with one of the axes.

Coterminal angles are angles in standard position that have the same terminal side.

The **reference angle** for an angle in standard position is the angle formed between the terminal side of the angle and the x-axis. Reference angles are always positive, and always < 90°.

An **arc** is a part of the circumference of a circle.

Arc length is the distance from one point on the circumference of a circle to another point on the circumference, "traveling" along the edge of the circle.

An **intercepted arc** is an arc in the interior of an angle.

A **minor arc** is an arc that measures less than 180°.

A **major arc** is an arc that measures greater than 180°.

A **radius** is the segment from the center to a point on the circle.

A **chord** is a segment that joins two points of the circle.

A **diameter** is a chord that contains the center of the circle.

A **tangent** is a line in the plane of the circle that intersects the circle in exactly one point. If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of tangency.

A **secant** is a line that intersects a circle in two points.

An **inscribed angle** is an angle with its vertex "on" the circle, formed by two intersecting chords. The measure of an inscribed angle is half of the measure of its intercepted arc. Inscribed angles on a diameter are right angles.

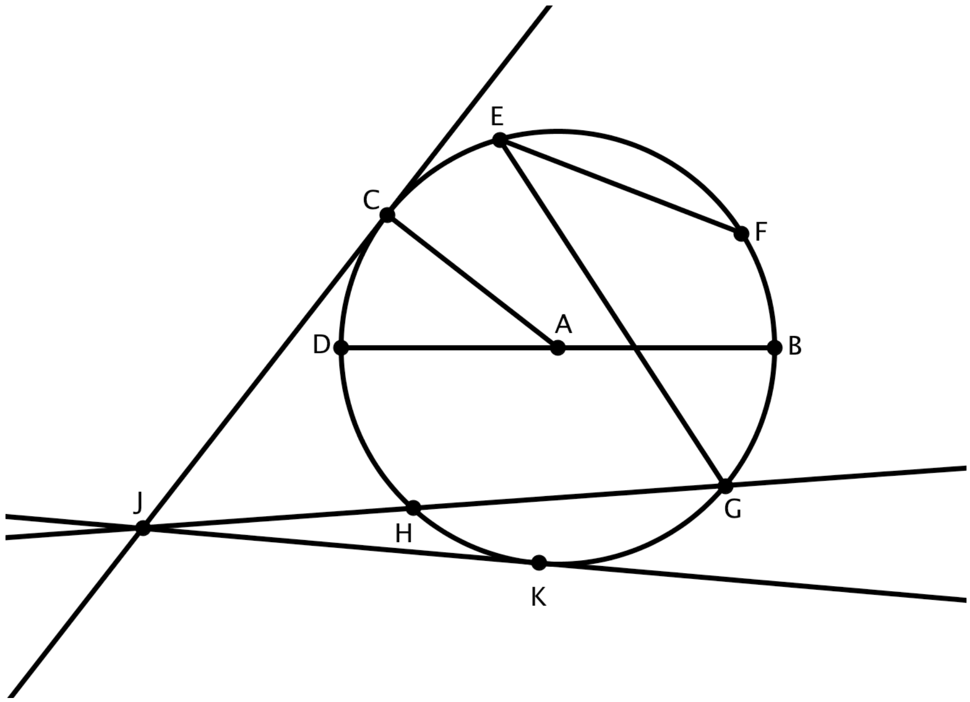
A **central angle** (of a circle) is an angle whose vertex is the center of the circle. The measure of a central angle is equal to the measure its intercepted arc.

The **area of a sector** is the area of the interior formed by a central angle and its intercepted arc.

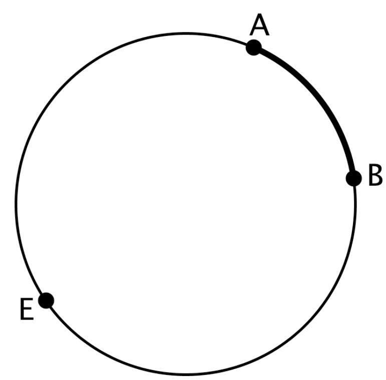
A **semicircle** is an arc of a circle that measures 180^o. An angle inscribed in a semicircle = 90^o.

An **outside angle** is an angle whose vertex lies outside the circle and whose rays intersect the circle. The measure of an outside angle is half the difference of the two intercepted arcs.

The **initial side** of an angle is the ray that remains fixed; the angle of rotation begins from this ray.



More on Arcs



Arcs are named by their endpoints. The arc at right would be called "arc AB". or "arc BA", the order of the endpoints does not matter. As a short hand this can be written as the letters AB with a curving line above them.

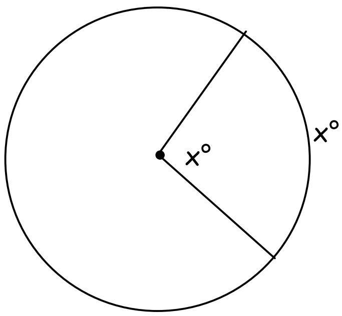
Example: $\overset{\frown}{AB}$ which is read "arc AB".

Notice that this naming can be ambiguous. For example it may mean the major arc AB, where you go the long way around the bottom of the circle. Unless stated otherwise, it always means the minor arc - the shortest of the two.

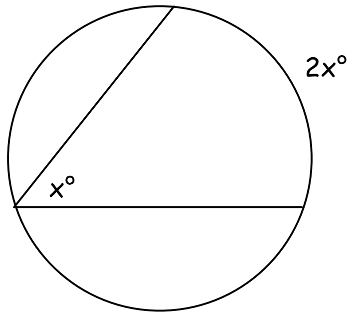
If you want to indicate the major arc, add an extra point and use three letters in the name. For example in the diagram on the right the major arc is indicated by

$\overset{\frown}{AEB}$ which is the long arc from A to B going around the bottom via E.

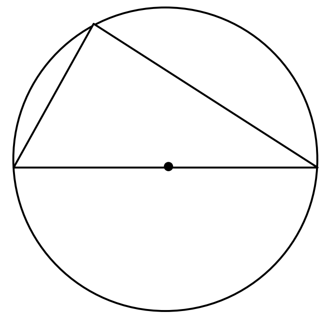
CIRCLE RELATIONSHIPS



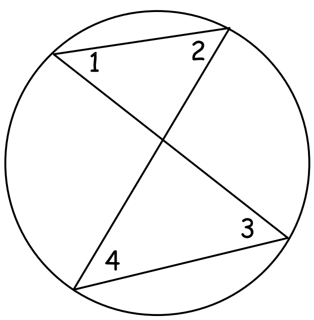
Central ∠ =



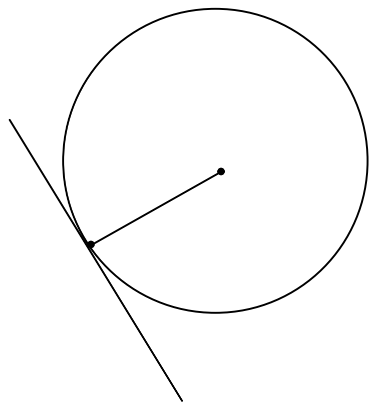
Inscribed ∠ =



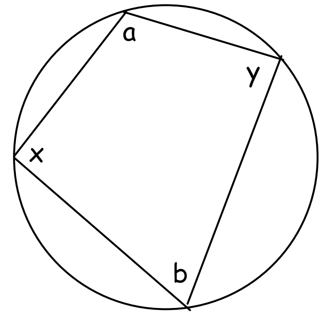
∠ inscribed in a semi-circle is a ______________



Inscribed ∠'s intercepting the same arc are ______



Tangent and a segment containing the center form a _____________ at the point of tangency



Opposite ∠'s in an inscribed quad. are _____________

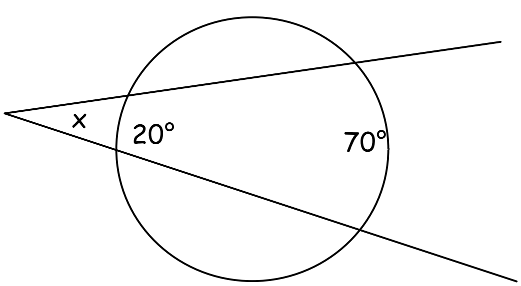
a + b = ______

x + y = ______

OUTSIDE ANGLES: Angles formed by segments outside of a circle

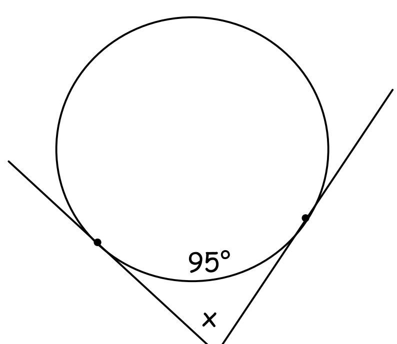
$$M\text{ (outside angle)} = \frac{\textit{outer arc} - \textit{inner arc}}{2}$$

CASE 1 : 2 SECANT SEGMENTS



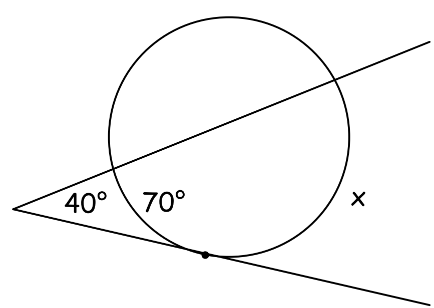
x= _______

CASE 2: 2 TANGENTS



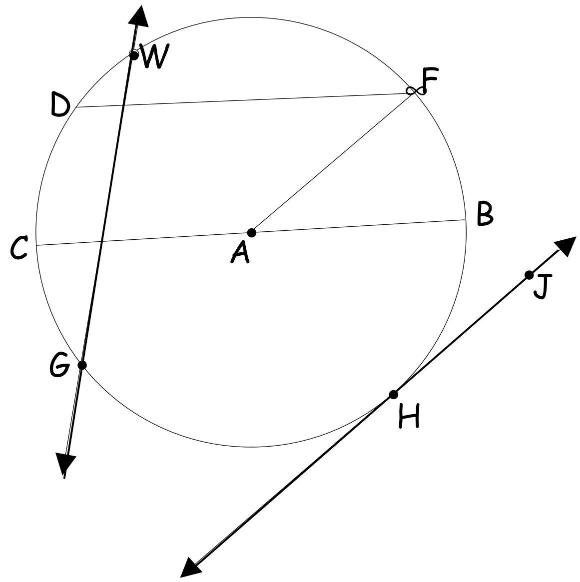
x= _______

CASE 3: 1 SECANT/ 1 TANGENT



x= _______

Definitions, Angle, Arc Worksheet



For CIRCLE A Identify the following:

1. $\overline{AB}$ ________________________

2. $\overline{DF}$ _______________________

3. $\overleftrightarrow{WG}$ ________________________

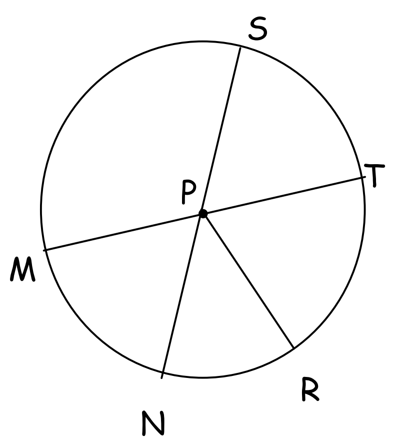
4. $\overleftrightarrow{HJ}$ ________________________

5. point H _______________________

6. $\overline{CB}$ __________________________

7. point A _______________________

8. Circle P with $m\angle NPR = 29°$ and $m\angle SPT = 51°$
   Determine the **degree of each arc** and the **type (major,minor,semi-circle).**



a) $m\overset{\frown}{NR}$ = ____________________ type _____________________

b) $m\overset{\frown}{ST}$ = _____________________ type _____________________

c) $m\overset{\frown}{TSR}$ = ____________________ type _____________________

d) $m\overset{\frown}{MST}$ =____________________ type _____________________

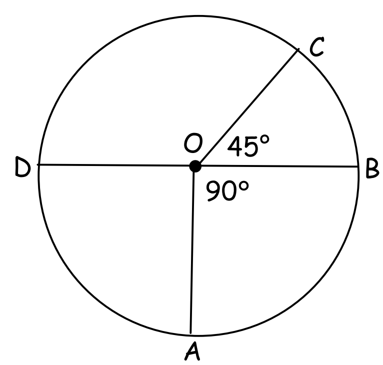
9 - 12 refer to ⊙ O. Find the measure of each arc.

9. $m\overset{\frown}{AB}$ __________

10. $m\overset{\frown}{CD}$ _____________

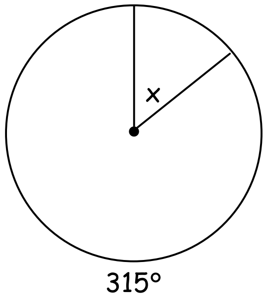
11. $m\overset{\frown}{AC}$ __________

12. $m\overset{\frown}{ADC}$ ____________



Find the value of x.

13.



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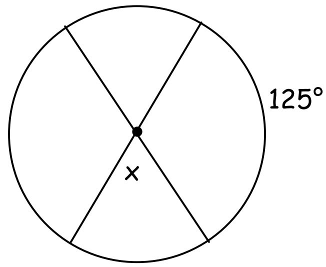

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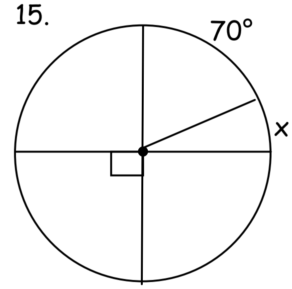
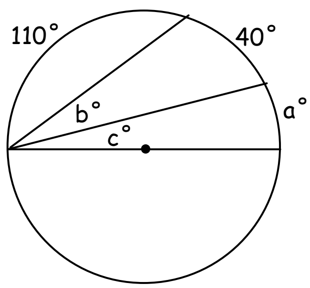


16. At ten o'clock the hands of a clock form an angle of ________ degrees.

17. At seven o'clock the hands of a clock form an angle of ________degrees.

18.

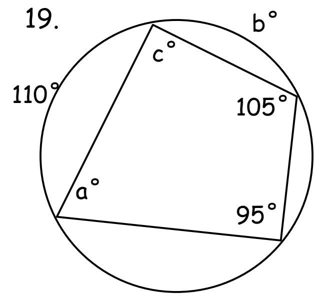


a = __________

b = __________

c = __________

19.

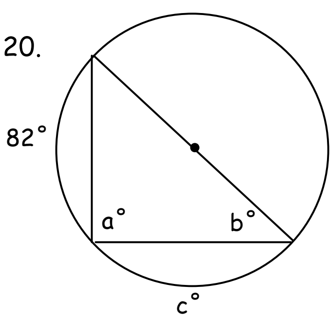


a = __________

b = __________

c = __________

20.

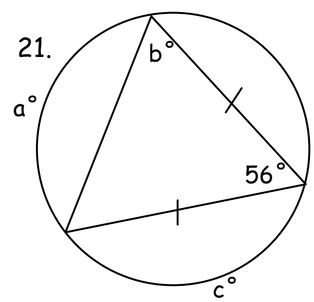


a = __________

b = __________

c = __________

21.

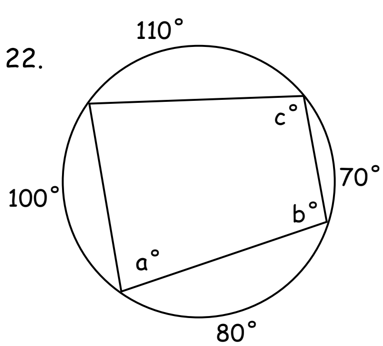


a = __________

b = __________

c = __________

22.

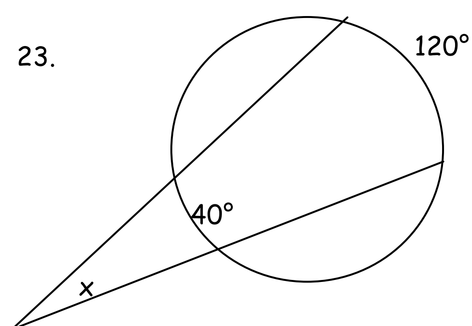


a = __________

b = __________

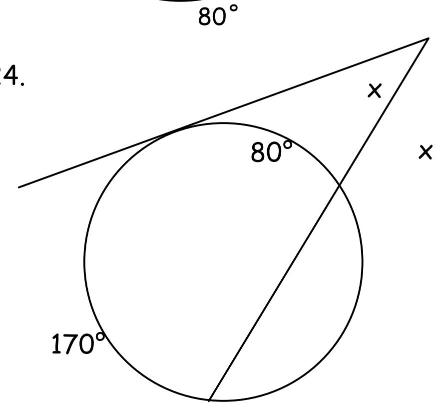
c = __________

23.



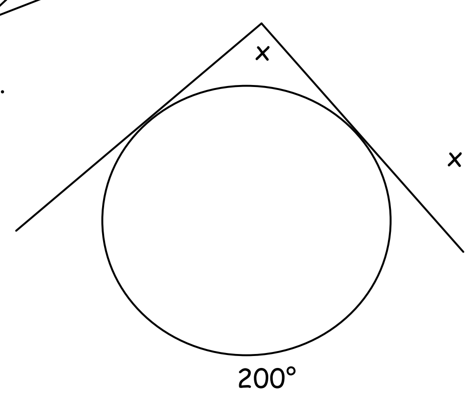
x = __________

24.



x = __________

25.

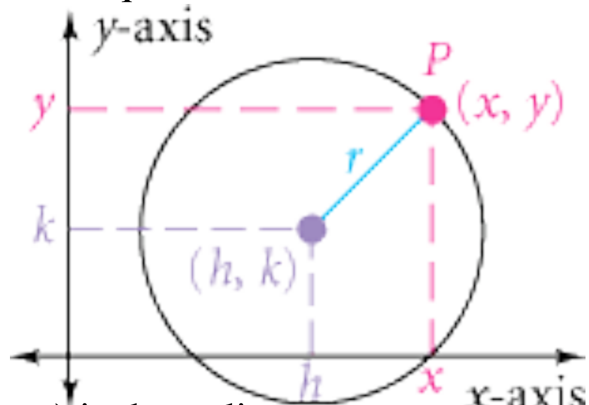


x = __________

CIRCLES

A circle is the set of all points in a plane that are a distance r from a given point, called the center. The distance r is the radius of the circle.

You can use the distance formula to find an equation of a circle with a radius r and a center at the point (h, k).



Let (x, y) be any point on the circle.
Let (h, k) be the point at the center of the circle.
Let r be the radius.

The distance from the center (h, k) to (x, y) is the radius.

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$	Distance Formula
$\sqrt{(x-h)^2 + (y-k)^2} = r$	Distance from (h,k) to (x,y) = r
$(x-h)^2 + (y-k)^2 = r^2$	Square each side to get the equation of a circle.

STANDARD FORM of the equation of a circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

GENERAL FORM of the equation of a circle:

$$x^2 + y^2 + Dx + Ey + F = 0$$

- The equation of the circle with a center at the origin is $x^2 + y^2 = r^2$
- To convert from standard to general form – multiply it out.
- To convert from general to standard form – complete the square.
- All circles are similar.

Example 1: Write an equation of a circle with center (–4, 3) and radius 4.

$(x - h)^2 + (y - k)^2 = r^2$ Use the standard form of the equation of a circle.

$(x - (-4))^2 + (y - 3)^2 = 4^2$ Substitute −4 for *h*, 3 for *k*, and 4 for *r*.

$(x + 4)^2 + (y - 3)^2 = 16$ Simplify.

An equation for the circle is $(x + 4)^2 + (y - 3)^2 = 16$.

Example 2: Find the center and radius of the circle with equation.
$(x - 16)^2 + (y + 9)^2 = 144$.

$(x - h)^2 + (y - k)^2 = r^2$ Write the standard form.

$(x - 16)^2 + (y + 9)^2 = 144$ Write the equation.

$(x - 16)^2 + (y - (-9))^2 = 12^2$ Rewrite the equation in standard form.

$h = 16 \quad k = -9 \quad r = 12$ Find *h*, *k*, and *r*.

The center of the circle is (16, –9). The radius is 12.

Example 3: Convert from standard form to general form by multiplying it out.

$(x + 6)^2 + (y - 2)^2 = 4$ **Standard Form**

$(x + 6)(x + 6) + (y - 2)(y - 2) = 4$ **Multiply it out**

$x^2 + 12x + 36 + y^2 - 4y + 4 = 4$ **Combine and move terms to look**

$x^2 + y^2 + 12x - 4y + 40 = 4$ **like:** $x^2 + y^2 + Dx + Ey + F = 0$

$x^2 + y^2 + 12x - 4y + 36 = 0$ **General Form**

Example 4: Convert from general from to standard form by completing the square. Then find the center and radius.

$x^2 + y^2 + 8y - 10x + 16 = 0$ **General Form**

$x^2 - 10x + y^2 + 8y = -16$ **Group the x terms, group the y terms, move constants to the other side.**

$(x - 5)^2 + (y + 4)^2 = -16 + 25 + 16$ **Complete the square for x and y.**

$(x - 5)^2 + (y + 4)^2 = 25$ **Simplify.**

The center is (5, -4) and the radius is 5.

Practice:

1. Write an equation of a circle with a radius of 8 and a center of (4, 3).

2. Write an equation of a circle with a radius of 6 and a center of (-3, -8).

3. Write the equation of a circle given the center is (2, -5) with a radius of 7.

4. Find the center and radius. $(x-1)^2+(y+4)^2=81$

5. Find the radius and center of: $(x-5)^2+(y-2)^2=20$

6. Convert to general form. $(x-2)^2+(y+3)^2=9$

7. Convert to general form. $(x+4)^2+(y-2)^2=1$

8. Convert to standard form, then find the center and radius. $x^2-8x+y^2+11=0$

9. Convert to standard form, then find the center and radius. $x^2+y^2+4x-6y=-4$

10. Convert to standard form, then find the center and radius. $3x^2=9-3y^2-6y$

Example 5: Write an equation of a circle whose diameter has endpoints (3,5) and (6,1).

midpoint of diameter is the center of the circle; diameter = 2 • radius,

Use midpoint formula to find the center: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Center: $\left(\dfrac{3+6}{2}, \dfrac{5+1}{2}\right) = \left(\dfrac{9}{2}, 3\right)$

Use distance formula to find the diameter: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Diameter: $d = \sqrt{(6-3)^2 + (1-5)^2} = \sqrt{9+16} = 5$

Radius is ½ the diameter: $r = \dfrac{5}{2}$

Use the center and radius to write the equation in standard form:

$$\left(x - \frac{9}{2}\right)^2 + (y-3)^2 = \frac{25}{4}$$

Practice:

11. Write an equations of a circle whose diameter has endpoints (2,8) and (2,-2) .

12. Write an equation for a circle with center (7,7) that passes through (12,9)

13. Write an equation for a circle with center (-4, -3) that is tangent to the x-axis.

Equation of Circles Worksheet

I. Write the equation of the circle that satisfies each set of conditions.

1. Center (0, 3), radius is 7 units

2. Center $(-8, 7)$, radius is $\frac{1}{2}$ units

3. Center $(-1, -5)$, radius is 2 units.

II. Find the center and radius of the circle with the given equation.

4. $x^2 + (y + 2)^2 = 4$

5. $x^2 + y^2 = 144$

6. $(x - 3)^2 + (y - 1)^2 = 25$

7. $(x + 3)^2 + (y + 7)^2 = 81$

8. $(x - 3)^2 + y^2 = 16$

9. $x^2 + y^2 + 6y = -50 - 14x$

10. $x^2 + y^2 + 2x - 10 = 0$

11. $x^2 + y^2 - 18x - 18y + 53 = 0$

12. $x^2 + y^2 + 2x + 4y = 9$

Equations of Circles Worksheet 2 **Honors**

Write the equation of the circle that satisfies each set of conditions.

1. Endpoints of a diameter at $(-5,2)$ and $(3,6)$

2. Endpoints of a diameter at $(11,18)$ and $(-13,-19)$

3. Center at $(8,-9)$, passes through $(21,22)$

4. Center at $(-8,-7)$, tangent to y-axis

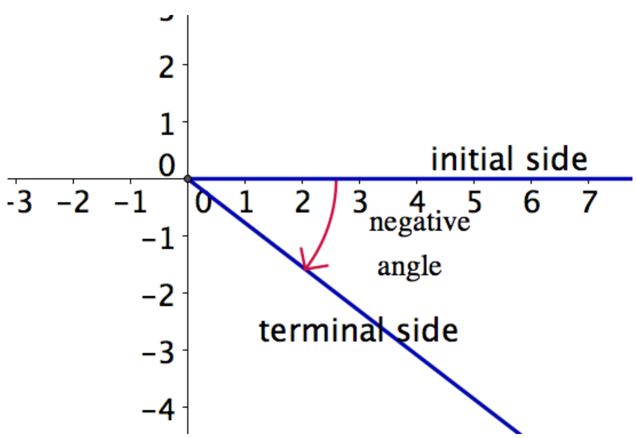
5. Center at $(4,2)$, tangent to the x-axis

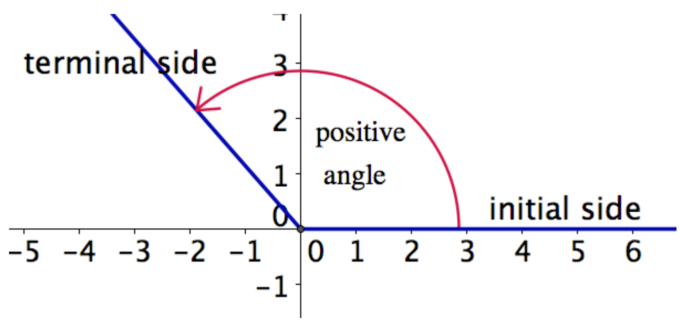
Trigonometry: the branch of mathematics that studies relationships involving lengths and angles of triangles.

Angle: generated by the rotation of two rays that share a fixed endpoint

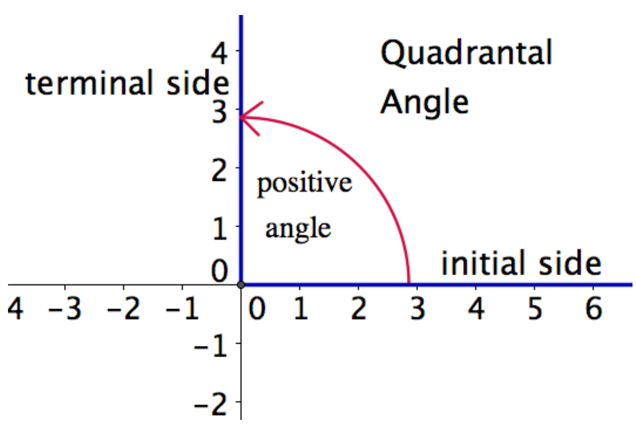
- *Initial side:* fixed ray
- *Terminal side:* ray that rotates away from the initial side
  - Positive angle – counterclockwise rotation
  - Negative angle – clockwise rotation

An angle with its vertex at the origin and its initial side along the positive x-axis is said to be in **standard position**.





If the terminal side of an angle in standard position coincides with one of the axes, the angle is called a **quadrantal angle**.



Units for measuring angles: **HONORS**

Degrees: A circle is divided into 360 equal degrees, so that a right angle is 90°.

- 1 degree = 60 minutes (1° = 60')
- 1 minute = 60 seconds (1' = 60")

Examples:

1. Change 29° 45' 26" to a decimal number of degrees.

$$29^\circ + 45'\left(\frac{1^\circ}{60'}\right) + 26''\left(\frac{1'}{60''}\right)\left(\frac{1^\circ}{60'}\right) = 29.757^\circ$$

To convert to decimal degrees in the calculator, enter the degrees, minutes, seconds in the calculator including the units (° ' ") then hit Enter.

- Degree Symbol: 2^{nd} Apps (angle) #1
- Minute Symbol: 2^{nd} Apps (angle) #2
- Seconds Symbol: Alpha +

2. Convert 34.624° to degrees, minutes, and seconds.

$$\begin{aligned}
34.624^\circ &= 34^\circ + .624^\circ \\
&= 34^\circ + .624^\circ\left(\frac{60'}{1^\circ}\right) \\
&= 34^\circ + 37.44' \\
&= 34^\circ + 37' + .44'\left(\frac{60''}{1'}\right) \\
&= 34^\circ + 37' + 26'' \\
&= 34^\circ 37' 26''
\end{aligned}$$

- To round to the nearest degree minutes and seconds in the calculator go to 2^{nd} Apps (angle) #4 DMS. This will take a degree measure that is in decimal form and put in degrees minutes and seconds.

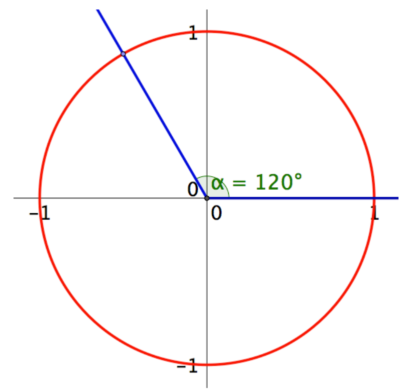
Units for measuring angles:

Degrees: A circle is divided into 360 equal degrees, so that a right angle is 90°.

Radians: The radian measure of an angle in standard position is defined as the length of the corresponding arc on the unit circle.

A **unit circle** is a circle of radius 1 whose center is at the origin. The equation of the unit circle is $x^2+y^2=1$.

The circumference of a unit circle is 2π**,** so the radian measure of an angle of one full revolution (or 360°) is 2π.



Degree/Radian Conversion:

- 180° = π radians
- radians are usually written in term of π
- the degree symbol (°) is always written, the word "*radians*" is usually omitted

One radian is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.

Examples:

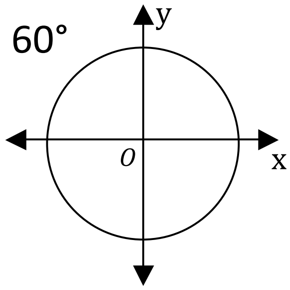
1. Change 30° to radians: $30° \times \left(\dfrac{\pi}{180°}\right) = \dfrac{\pi}{6}$

2. Change $\dfrac{3\pi}{4}$ radians to degrees: $\dfrac{3\pi}{4} \times \left(\dfrac{180°}{\pi}\right) = 135°$

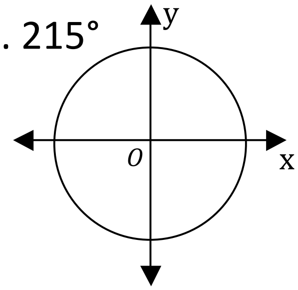
Practice:

Sketch each angle in standard position.

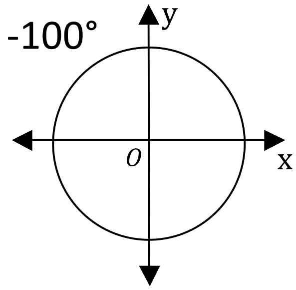
1. $60°$



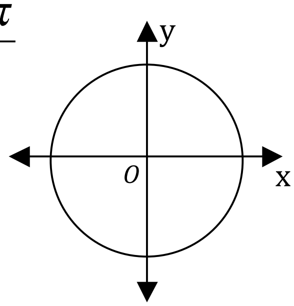
2. $215°$



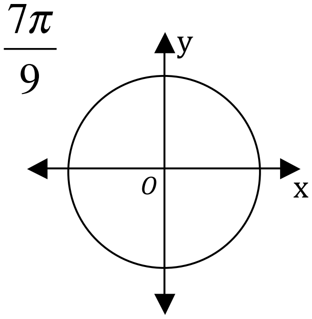
3. $-100°$



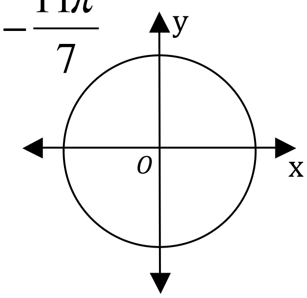
4. $\frac{5\pi}{4}$



5. $\frac{7\pi}{9}$



6. $-\frac{11\pi}{7}$



If each angle has the given measure and is in standard position, determine the quadrant in which its terminal side lies.

7. $735°$ 8. $-132°$ 9. $248°$ 10. $-305°$ 11. $465°$

12. $\frac{7\pi}{8}$ 13. $-\frac{3\pi}{4}$ 14. $\frac{9\pi}{5}$ 15. $-\frac{13\pi}{6}$ 16. $\frac{5\pi}{14}$

Convert degrees to radians:

17. $45°$ 18. $150°$ 19. $72°$ 20. $270°$ 21. $99°$

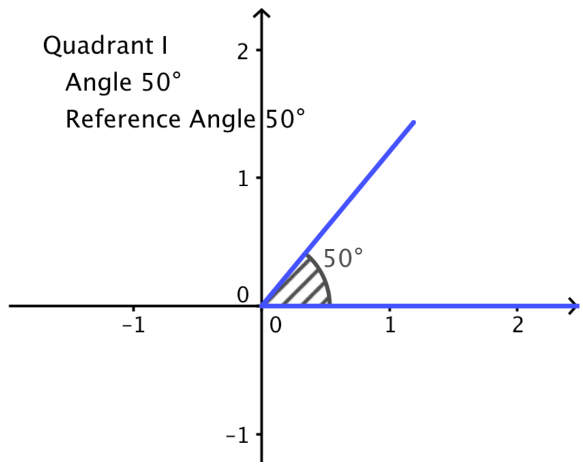
Convert radians to degrees:

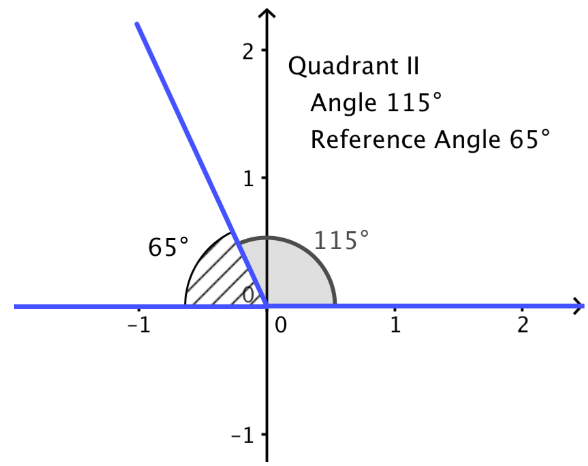
22. $\frac{\pi}{2}$ 23. $\frac{3\pi}{4}$ 24. $\frac{2\pi}{5}$ 25. $\frac{5\pi}{6}$ 26. $\frac{3\pi}{14}$

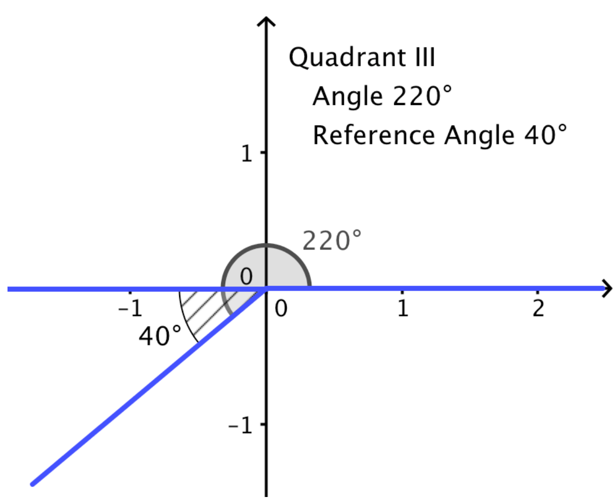
Two angles in standard position are called **coterminal angles** if they have the same terminal side. Angles with the same terminal side differ only in the number of revolutions.

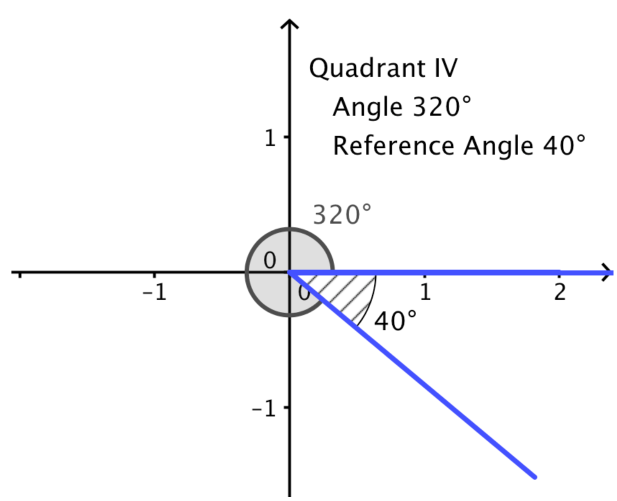
- Angles differing in radian measure by multiples of 2π are equivalent.
- Angles differing in degree measure by 360° are equivalent.
- Every angle has infinitely many coterminal angles.

Reference Angle: the acute angle (< 90°) formed by the terminal side of the given angle and the x-axis. Reference angles are <u>always</u> positive and <u>always</u> less than 90°.









Practice:

Find one positive angle and one negative angle that are coterminal with each angle.

1. 110° 2. -424° 3. $\frac{7\pi}{8}$ 4. $\frac{-9\pi}{5}$

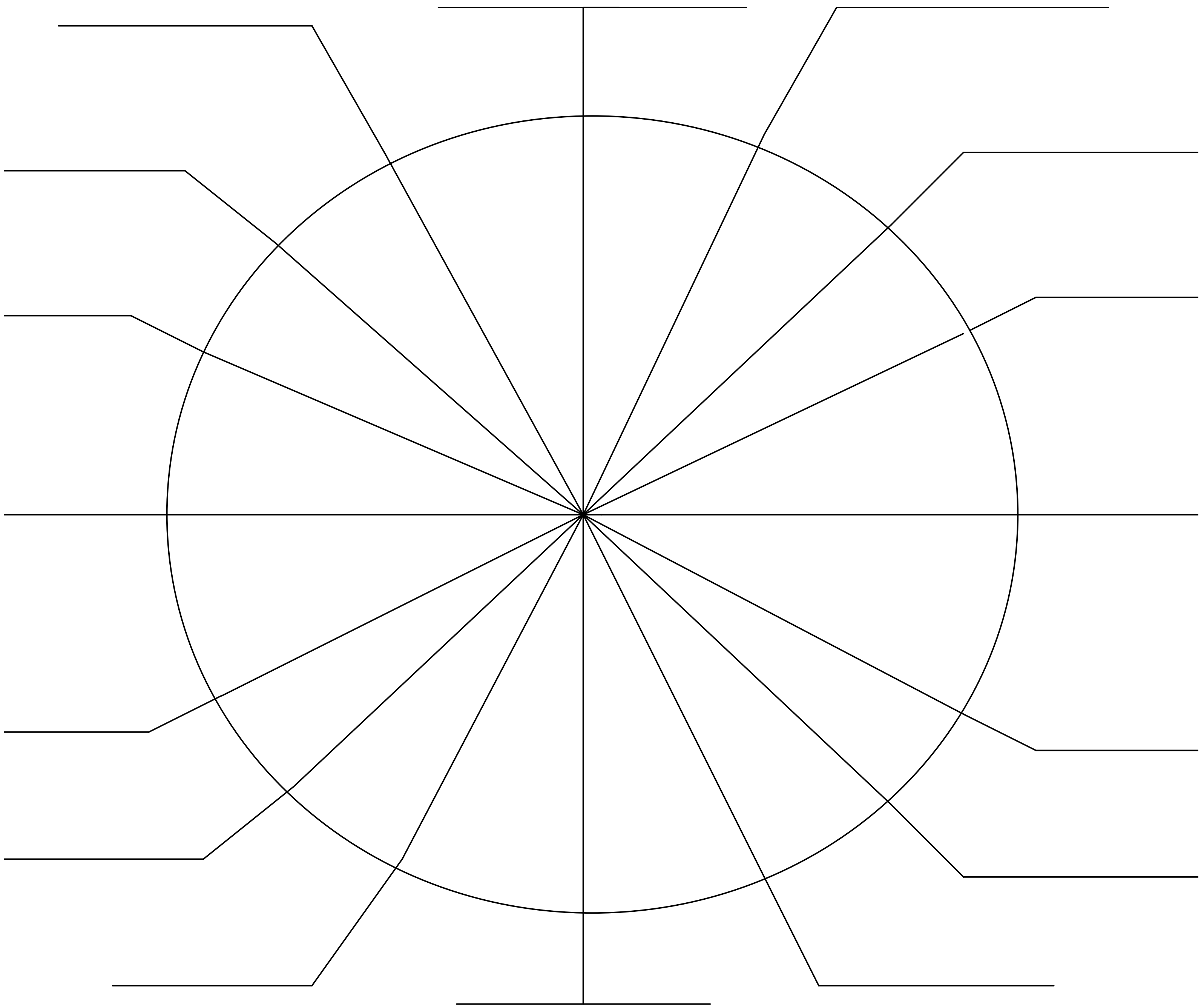
Find the reference angle for each angle with the given measure.

5. -35° 6. 245° 7. -510° 8. 85°

9. $\frac{7\pi}{4}$ 10. $\frac{-8\pi}{5}$ 11. $\frac{7\pi}{6}$ 12. $\frac{4\pi}{9}$

NOW… label the angles on the Unit Circle in degrees and radians.

Unit Circle



Angles and their Measures Worksheet

If each angle has the given measure and is in standard position, determine the quadrant in which its terminal side lies.

1. $\frac{7\pi}{12}$ 2. $-\frac{2\pi}{3}$ 3. $371°$ 4. $\frac{14\pi}{5}$

5. $-156°$ 6. $1000°$ 7. $332°$ 8. $-240°$

Change each degree measure to radian measure in terms of π.

9. $36°$ 10. $-250°$ 11. $-145°$ 12. $6°$

13. $870°$ 14. $18°$ 15. $-820°$ 16. $345°$

Change each radian measure to degree measure.

17. -1 18. 4π 19. -2.56 20. 12.85

21. $\frac{3\pi}{16}$ 22. $-\frac{7\pi}{9}$ 23. $\frac{13\pi}{30}$ 24. $-\frac{17\pi}{3}$

Find one positive angle and one negative angle that are coterminal with each angle.

25. $70°$ 26. $-\frac{2\pi}{5}$ 27. $-300°$ 28. $\frac{3\pi}{4}$

Find the reference angle for each angle with the given measure.

29. $-20°$ 30. $160°$ 31. $-545°$ 32. $300°$

33. $\frac{10\pi}{3}$ 34. $-\frac{5\pi}{8}$ 35. $-\frac{\pi}{4}$ 36. $-\frac{7\pi}{3}$

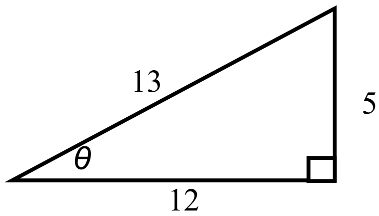
Remember **SOHCAHTOA?**

$$\sin = \frac{opposite}{hypotenuse} \qquad \cos = \frac{adjacent}{hypotenuse} \qquad \tan = \frac{opposite}{adjacent}$$

$\sin\theta =$

$\cos\theta =$

$\tan\theta =$



- SOHCAHTOA <u>only</u> applies to right triangles
- "opposite" and "adjacent" are relative to the angle of interest
- There are SIN, COS, and TAN buttons on the calculator.
- The calculator has 2 Modes for angles: Degrees and Radians
- The Pythagorean Theorem applies to right triangles

HONORS:

There are 3 additional trigonometric ratios or functions:

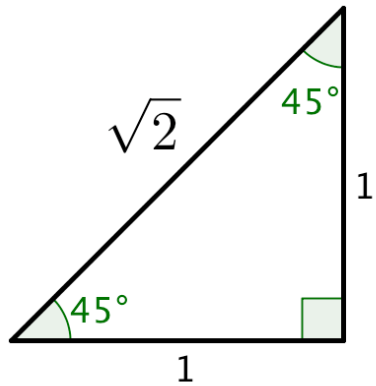
$$\csc = \frac{hypotenuse}{opposite} \quad or \quad \csc = \frac{1}{\sin}$$

$$\sec = \frac{hypotenuse}{adjacent} \quad or \quad \sec = \frac{1}{\cos}$$

$$\cot = \frac{adjacent}{opposite} \quad or \quad \cot = \frac{1}{\tan}$$

Remember the ‘special’ triangles?

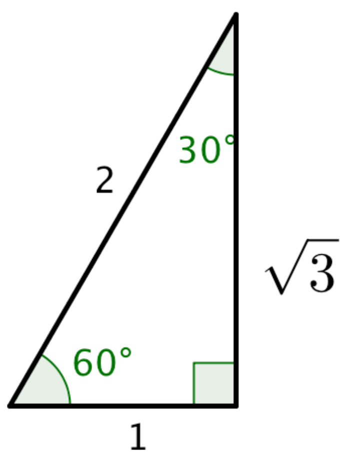
- **Isosceles Right Triangle:** 45° – 45° – 90°



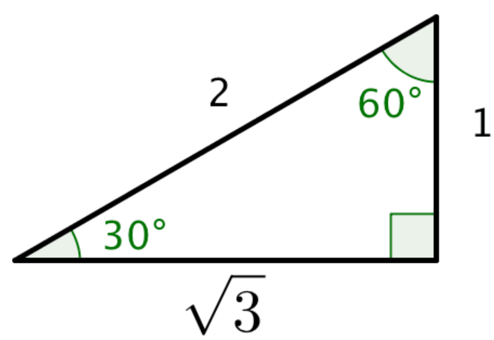
For 45-45-90 remember 1, 1, $\sqrt{2}$

cos45°= sin45°= tan45°=

- **Equilateral Triangle:** 30° – 60° – 90°



For 30-60-90 remember 1, 2, $\sqrt{3}$



cos60°= sin60°= tan60°=

cos30°= sin30°= tan30°=

Remember:

- The smallest side is opposite the smallest angle
- The largest side is opposite the largest angle

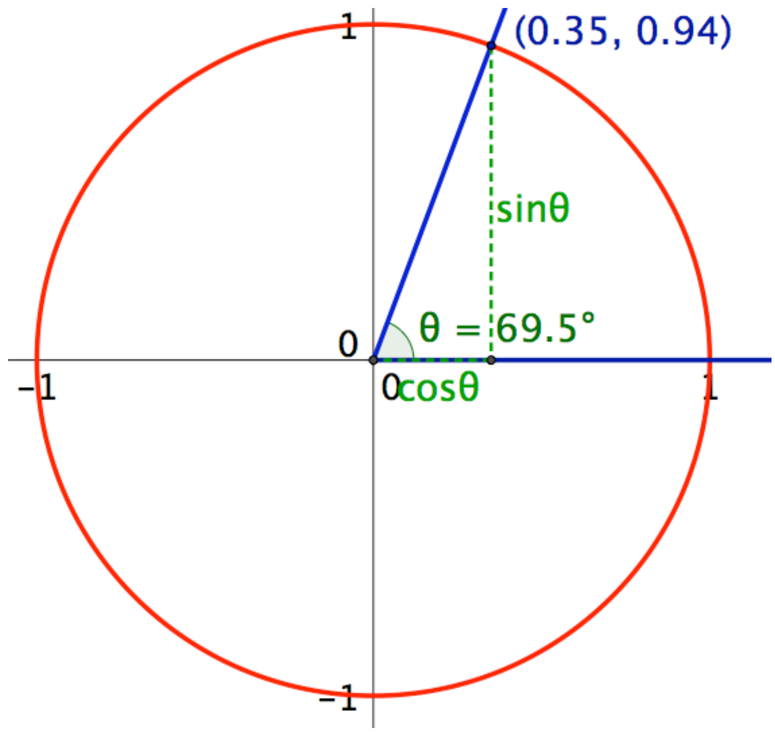
You must know the sin, cos, and tan for the special angles.
Either:

- Be able to create them from the triangles
  -OR-
- Memorize the table (or how to make it)

	30°	**45°**	**60°**
sin			
cos			
tan			
HONORS			
csc			
sec			
cot			

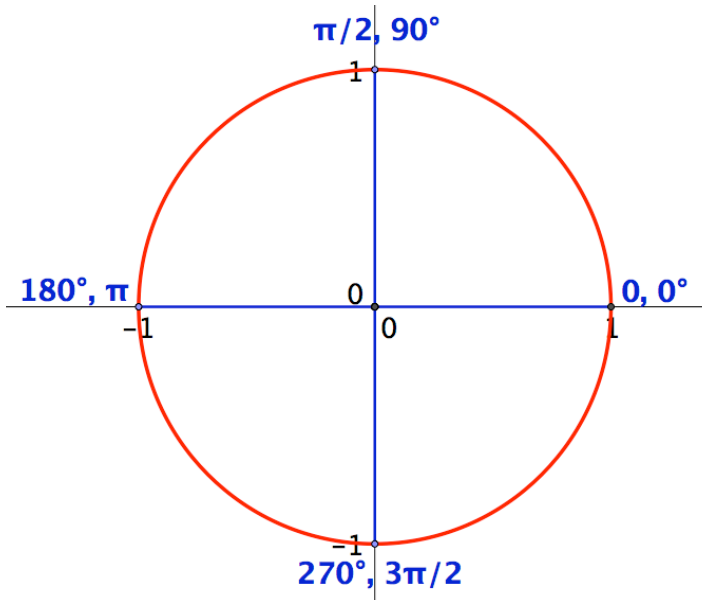
Definition of Sine and Cosine:

If the terminal side of an angle θ in standard position intersects the unit circle at point (x,y) then cosθ=x and sinθ=y



What about SOHCAHTOA?

Trigonometric Functions of Quadrantal Angles:



				HONORS		
Angle	**sin**	**cos**	tan	csc	sec	cot
0°						
90° or $\frac{\pi}{2}$						
180° or π						
270° or $\frac{3\pi}{2}$						
360° or 2π						

NOW…label sin and cos (as ordered pairs) for 30°, 45°, 60°, and the quadrantal angles on the Unit Circle.

Finish labeling sin and cos (as ordered pairs) for the remaining angles on the Unit Circle…

Complete the chart (No Calculator)

							Honors		
Degrees	Radians	Ref Angle or ordered pair	Quadrant	sin	cos	tan	csc	sec	cot
0°			X						
30°									
60°									
90°			X						
135°									
150°									
180°			X						
225°									
240°									
270°			X						
300°									
330°									

Unit Circle Worksheet

Complete the chart (No Calculator)							Honors		
Degrees	Radians	Ref Angle or ordered pair	Quadrant	sin	cos	tan	csc	sec	cot
	$\frac{\pi}{4}$								
	$\frac{7\pi}{6}$								
	$\frac{5\pi}{3}$								
	$-\frac{3\pi}{4}$								
	-3π								
	$-\frac{7\pi}{6}$								

Find each exact value. Do not use a calculator.

1. $\sin 240°$ 2. $\cos 135°$ 3. $\tan 45°$ 4. $\sin 300°$ 5. $\cos 210°$

6. $\tan 330°$ 7. $\tan 60°$ 8. $\sin -150°$ 9. $\sin\frac{11\pi}{6}$ 10. $\cos\frac{2\pi}{3}$

11. $\tan\frac{5\pi}{4}$ 12. $\sin\frac{3\pi}{2}$ 13. $\cos\frac{5\pi}{6}$ 14. $\tan\frac{4\pi}{3}$ 15. $\cos\frac{7\pi}{4}$

Use a calculator to approximate each value to four decimal places.
(Be sure your calculator is in the correct MODE, i.e. degrees vs. radians)

16. $\tan(-75°)$ 17. $\sin 634°$ 18. $\cos 127°$ 19. $\tan 311°$

20. $\sin\frac{7\pi}{5}$ 21. $\cos\frac{2\pi}{7}$ 22. $\cos 4.28$ 23. $\tan 0.23$

HONORS (no calc): 24. $\csc 330°$ 25. $\sec 240°$ 26. $\csc\frac{5\pi}{4}$ 27. $\cot\frac{5\pi}{6}$

HONORS (calc): 28. $\sec 138°$ 29. $\csc 266°$ 30. $\sec\frac{3\pi}{7}$ 31. $\cot\frac{13\pi}{11}$

Complete the chart (without a Calculator) **HONORS**

Given Angle	Reference Angle (or ordered pair)	Quadrant	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
1. 450°								
2. 90°								
3. 120°								
4. 900°								
5. 300°								
6. $\frac{7\pi}{3}$								
7. $-\frac{7\pi}{2}$								
8. $\frac{9\pi}{4}$								
9. $\frac{11\pi}{6}$								
10. $-\frac{19\pi}{4}$								
11. 3π								

Find the values USING A CALCULATOR. Round to FOUR places. **Be careful of MODE***

______________________12. sin 710° ______________________13. cos 72°30'42"

______________________14. tan 115°40' ______________________15. sin 7.42°

______________________16. $cos\left(-\frac{4\pi}{9}\right)$ ______________________17. csc 167°28'15"

______________________18. $\sec\frac{8\pi}{11}$ ______________________19. $\cot\frac{-13\pi}{5}$

______________________20. $\sin\frac{23\pi}{9}$ ______________________21. $\sec\frac{-40\pi}{7}$

Degrees, Radians, Sin & Cos of Special Angles

Write each measure in radians. Express your answer in terms of π .

1. 30°
2. 90°
3. 300°
4. 450°
5. 225°
6. 180°
7. 330°
8. -315°
9. 110°
10. 40°

Write each measure in degrees.

1. 2π
2. $\frac{5\pi}{4}$
3. $\frac{\pi}{3}$
4. $\frac{7\pi}{6}$
5. $\frac{2\pi}{3}$
6. $\frac{13\pi}{4}$
7. $\frac{4\pi}{3}$
8. $\frac{11\pi}{6}$
9. $\frac{2\pi}{9}$
10. $\frac{3\pi}{2}$

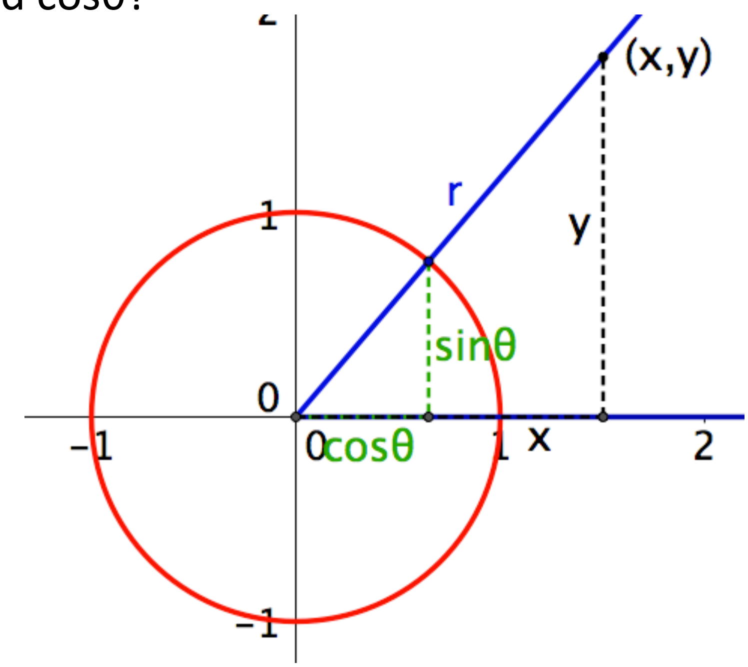
Find the exact value of the sine, cosine, and tangent of each angle. **Do not give decimals!**

1. π
2. $\frac{\pi}{4}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{6}$
5. $\frac{\pi}{2}$
6. -2π
7. $\frac{-3\pi}{2}$
8. $-\pi$
9. $\frac{3\pi}{2}$
10. 6π

Find the exact coordinate of the point where the terminal side of the angle intersects the unit circle.

1. 0°
2. 90°
3. 60°
4. 180°
5. -270°
6. 270°
7. 360°
8. 45°
9. -180°
10. 30°

If you know a point on the terminal side of an angle θ, but that point does not lie on the unit circle, how can you find the sinθ and cosθ?



For any angle θ in standard position, a point (x,y) on it's terminal side, and $r = \sqrt{x^2 + y^2}$, the sine and cosine of θ are: $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$

thus… $\tan\theta = \frac{y}{x}$

Note:
r is always positive

A **Pythagorean triple** consists of three positive integers *a*, *b*, and *c*, such that $a^2 + b^2 = c^2$. A triple is commonly written (*a*, *b*, *c*).
Some Pythagorean triples:

(3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25) (20, 21, 29)

Practice

Find the values of the sine, cosine, and tangent functions of an angle θ in standard position if the given point lies on its terminal side.

1. (5,12)

x=5, y = 12

$$r=\sqrt{5^2+12^2}=\sqrt{169}=13$$

$$\sin\theta=\frac{12}{13} \qquad \cos\theta=\frac{5}{13} \qquad \tan\theta=\frac{12}{5}$$

2. (-3,4)

							HONORS		
ORDERED PAIR	**x**	**y**	**r**	sin Θ	cos Θ	tan Θ	csc Θ	sec Θ	cot Θ
$(-8,-15)$									
$(-24,7)$									
$(2,-3)$									

							HONORS		
QUADRANT	x	y	r	sin Θ	cos Θ	tan Θ	csc Θ	sec Θ	cot Θ
III						$\frac{5}{12}$			
IV				$\frac{-7}{25}$					
II					$\frac{-4}{5}$				

Point on Terminal Side Worksheet
Complete the charts.

							HONORS		
ORDERED PAIR	**x**	**y**	**r**	$\sin\Theta$	$\cos\Theta$	$\tan\Theta$	$\csc\Theta$	$\sec\Theta$	$\cot\Theta$
$(7,-24)$									
$(-20,-21)$									
$(-3,4)$									
$(3,2)$									

							HONORS		
QUADRANT	x	y	r	$\sin\Theta$	$\cos\Theta$	$\tan\Theta$	$\csc\Theta$	$\sec\Theta$	$\cot\Theta$
III				$-\frac{8}{17}$					
IV					$\frac{3}{8}$				
I					$\frac{3}{5}$				
II						$\frac{-12}{5}$			

1. Find $\sin\theta$ *if* $\cos\theta = \frac{8}{17}$, Quad I

2. Find $\cos\theta$ *if* $\sin\theta = -\frac{4}{5}$, Quad III

More Practice Worksheet

In which quadrant does the terminal side lie if the angle is in standard position?

1. -280° 2. 425° 3. $-\frac{3\pi}{4}$ 4. $\frac{17\pi}{6}$

Determine the coterminal angle between 0° & 360° for the given angle.

5. -225° 6. 520° 7. $-\frac{7\pi}{8}$ 8. $\frac{19\pi}{6}$

Express each angle in radians. Leave in terms of π.

9. 468° 10. -210°

Express each angle in degrees.

11. $\frac{5\pi}{6}$ 12. $\frac{-5\pi}{4}$

Find the reference angle for each angle. Your answer should be in the same units as the given angle.

13. -60° 14. 210° 15. $\frac{5\pi}{6}$ 16. $\frac{5\pi}{4}$ 17. $\frac{-7\pi}{8}$

If θ is an angle in standard position whose terminal side is in the given Quandrant, find the values of the following:

18. $\sin\theta = \frac{12}{13}$, Quad II x = ________ cosθ = ________ tanθ = _______

Honors: secθ= _______ cscθ = _______ cotθ =________

19. $\cos\theta = \frac{\sqrt{3}}{2}$, Quad I y = ________ sinθ = ________ tanθ =_______

Honors: secθ= _______ cscθ = _______ cotθ = ________

Length of a Circular Arc

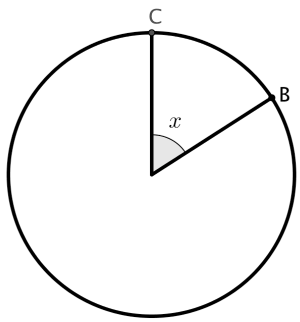
Arcs have two properties. They have a measurable curvature based upon the corresponding central angle (measure of arc = measure of central angle). Arcs also have a length as a portion of the circumference.

$$\frac{portion\ \ of\ \ circle}{whole\ \ circle} = \frac{central\ \ angle\ \ in\ \ \deg rees}{360°} = \frac{central\ \ angle\ \ in\ \ radians}{2\pi} = \frac{arc\ \ length}{circumference}$$

$$\frac{x°}{360°} = \frac{length\ \ \overset{\frown}{CB}}{2\pi r}$$

-or -

$$\frac{x\ \ (radians)}{2\pi} = \frac{length\ \ \overset{\frown}{CB}}{2\pi r}$$



Remember:

- circumference of a circle = $2\pi r$

For a central angle **θ** in radians, and arc length **s** - the proportion can be simplified to a formula:

$$\frac{\theta}{2\pi} = \frac{s}{2\pi r}$$

$$s 2\pi = \theta\, 2\pi r$$

$$s = \theta r$$

Length of an Arc: $\mathbf{s = r\theta}$
for θ in radians

Examples:

1) For a central angle of $\pi/6$ in a circle of radius 10 cm, find the length of the intercepted arc.

2) For a central angle of $4\pi/7$ in a circle of radius 8 in, find the length of the intercepted arc.

3) For a central angle of 40° in a circle of radius 6 cm, find the length of the intercepted arc.

4.) Find the degree measure to the nearest tenth of the central angle in a circle that has an arc length of 87 and a radius of 16 cm.

Area of a Sector

Sector of a circle: a region bounded by a central angle and the intercepted arc

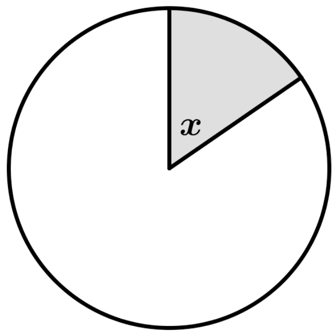
Sectors have an area as a portion of the total area of the circle.

$$\frac{portion\ \ of\ \ circle}{whole\ \ circle} = \frac{central\ \ angle\ \ in\ \ \deg rees}{360^\circ} = \frac{central\ \ angle\ \ in\ \ radians}{2\pi} = \frac{area\ \ of\ \ \sec tor}{area\ \ of\ \ circle}$$

$$\frac{x^\circ}{360^\circ} = \frac{area\ \ of\ \ \sec tor}{\pi r^2}$$

-or -

$$\frac{x\ \ (radians)}{2\pi} = \frac{area\ \ of\ \ \sec tor}{\pi r^2}$$



Remember:

- area of a circle = πr^2

For a central angle **θ** in radians, and area of sector **A**, the proportion can be simplified to a formula:

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

$$A2\pi = \theta\pi r^2$$

$$A = \frac{1}{2}\theta r^2$$

Area of a Circular Sector: $A=\frac{1}{2}r^2\theta$
for θ in radians

Examples:

5) Find the area of the sector of the circle that has a central angle measure of $\pi/6$ and a radius of 14 cm.

6) Find the area of the sector of the circle that has a central angle measure of 60° and a radius of 9 in.

HONORS

7) A sector has arc length 12 cm and a central angle measuring 1.25 radians Find the radius of the circle and the area of the sector.

Arc & Sector Worksheet

I. Given the radian measure of a central angle, find the length of its intercepted arc in terms of π in a circle of radius 10 cm.

1. $\frac{\pi}{6}$ 2. $\frac{\pi}{3}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{5}$

5. $\frac{3\pi}{5}$ 6. $\frac{4\pi}{7}$ 7. $\frac{\pi}{12}$ 8. $\frac{\pi}{24}$

II. Given the measurement of a central angle, find the measure of its intercepted arc in terms of π in a circle of diameter 60 in.

9. 10° 10. 60° 11. 42° 12. 50°

13. 72° 14. 110° 15. 35° 16. 65°

III. Given the measure of an arc, find the degree measure to the nearest tenth of the central angle if subtends in a circle of radius 16 cm.

17. 87 18. 5.6 19. 12 20. 25

21. 10.24 22. 7.9 23. 11 24. 6

IV. Find the area of each sector to the nearest tenth, given its central angle, and the radius of the circle.

25. $\theta = \frac{\pi}{4}, r = 14 \quad cm$ 26. $\theta = \frac{\pi}{6}, r = 12 \quad ft.$ 27. $\theta = \frac{5\pi}{12}, r = 10 \quad ft.$

28, $\theta = 54°, r = 6 \quad in$ 29. $\theta = 82°, r = 7.3 \quad km$ 30. $\theta = 45°, r = 9.75 \quad mm$

HONORS: (round answers to nearest tenth)

31. A sector has arc length of 6 cm and a central angle measuring 1.2 radians. Find the radius of the circle and the area of the sector.

32. A sector has arc length of 10 in and a central angle measuring 50°. Find the radius of the circle and the area of the sector.

Pythagorean Identities **HONORS**

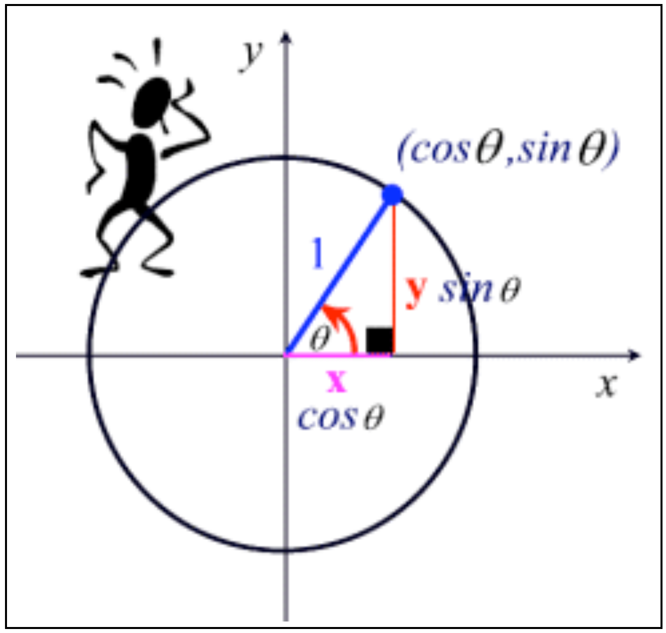
When studying the unit circle, it was observed that a point on the unit circle (the vertex of the right triangle) can be represented by the coordinates

$$(\cos\theta, \sin\theta).$$

Since the legs of the right triangle in the unit circle have the values of $\sin\theta$ and $\cos\theta$, the Pythagorean Theorem can be used to obtain

$$\sin^2\theta + \cos^2\theta = 1.$$

This well-known equation is called a **Pythagorean Identity.**



There are two additional Pythagorean Identities, which can be created from the first Pythagorean Identity.

$$\tan^2\theta + 1 = \sec^2\theta \quad and \quad 1 + \cot^2\theta = \csc^2\theta$$

Given the 1st Pythagorean Identity prove the 2nd and 3rd Pythagorean Identities.

Given: $\sin^2\theta + \cos^2\theta = 1$

Prove: $\tan^2\theta + 1 = \sec^2\theta$

Given: $\sin^2\theta + \cos^2\theta = 1$

Prove: $1 + \cot^2\theta = \csc^2\theta$

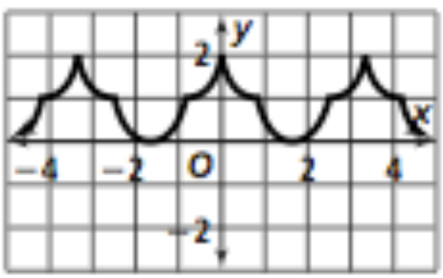
Periodic Functions

The graph of a **periodic function** shows a repeating pattern. The distance from 1 point on the graph to the point where the pattern begins repeating is called the **period**.

Periodic Function: repeats a pattern of y-values at regular intervals
Cycle: one complete pattern, a cycle may begin at any point on graph
Period: the horizontal (x) length of one cycle
Amplitude: half the distance between the maximum and minimum y-values of a function

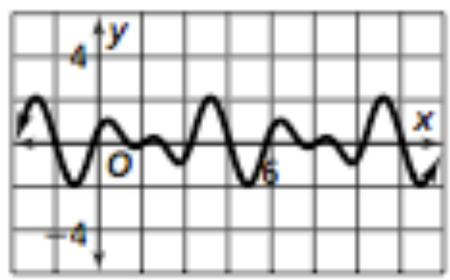
Examples:

State the period and amplitude of each function.



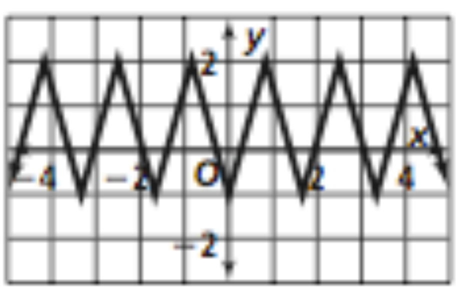
Period: ____________

Amplitude: _________



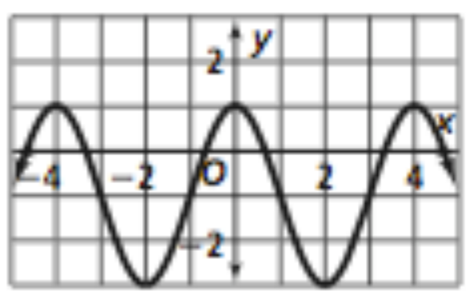
Period: _____________

Amplitude: __________



Period: ____________

Amplitude: _________

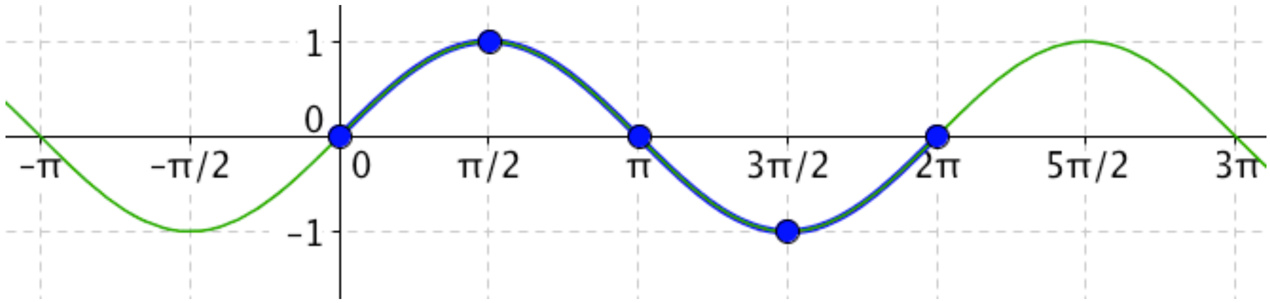


Period: _____________

Amplitude: __________

Graphing Sine

Sine Function: y = sinx (amplitude =1, period = 2π)



We will graph the angle measure (the x value) in radians.
To graph by hand we will find 5 key points. These points are the maximum, the minimum, and the x-intercepts. We will usually graph only 1 cycle.

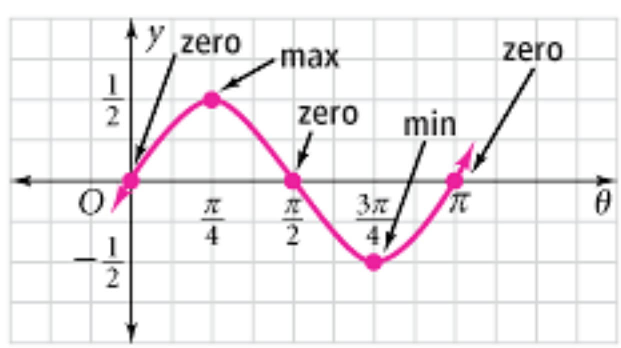
The graph of a sine function is called a **sine curve**.

For ***y = a sin bx*** with ***a≠ 0, b >0*** and **x in radians**:

- |*a*| is the amplitude of the function
- if *a* is negative the graph flips over the x-axis
- *b* is the number of cycles in the interval 0 to 2π
- $\frac{2\pi}{b}$ is the period of the function

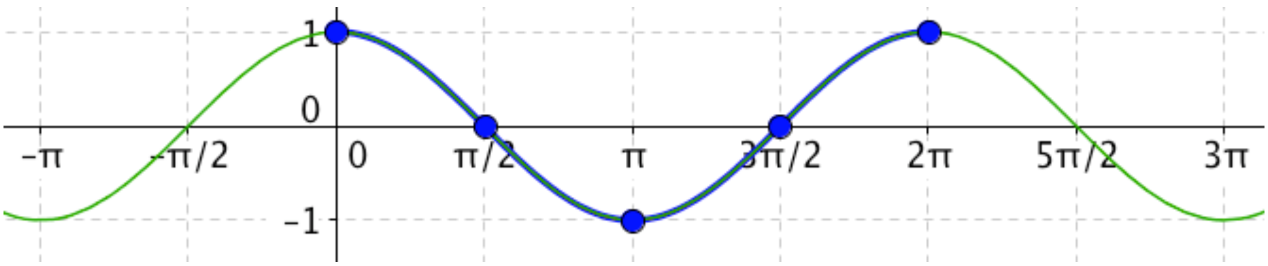
Example: Sketch one cycle of $y = \frac{1}{2}\sin 2x$

|*a*| = ½, so the amplitude is ½
b = 2 so there are 2 cycles from 0 to 2π
$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ so the period is π
Divide the period into fourths.
Using the values of the amplitude and period plot the pattern zero-max-zero-min-zero.



<u>Graphing Cosine</u>

Cosine Function: y = cos x (amplitude =1, period = 2π)

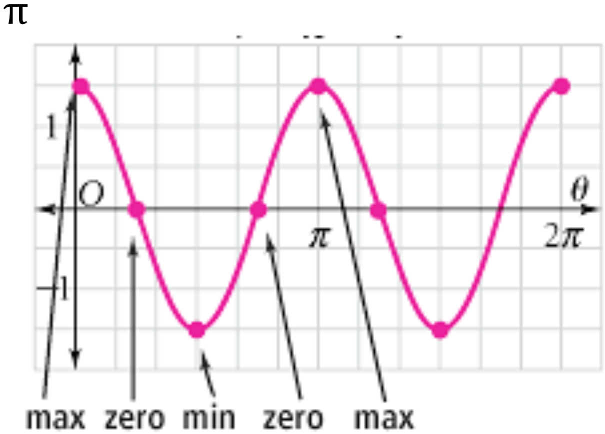


We will graph the angle measure (the x value) in radians.
To graph by hand we will find 5 key points. These points are the maximum, the minimum, and the x-intercepts. We will usually graph only 1 cycle.

For ***y = a cos bx*** with ***a≠ 0, b >0*** and **x in radians**:

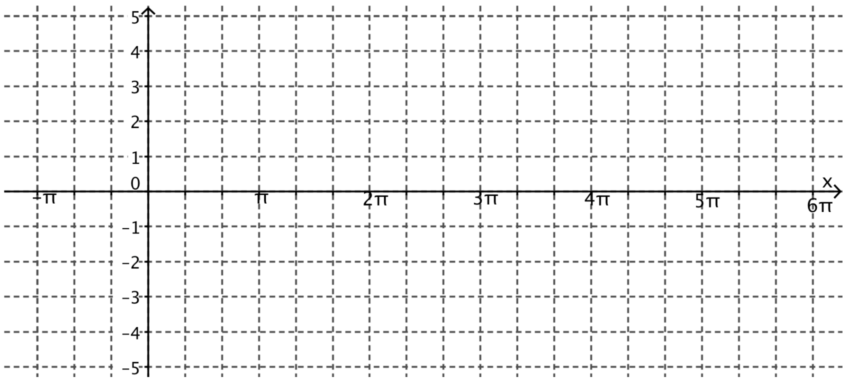
- $|a|$ is the amplitude of the function
- if a is negative the graph flips over the x-axis
- b is the number of cycles in the interval 0 to 2π
- $\frac{2\pi}{b}$ is the period of the function

Example: Sketch one cycle of $y = 1.5\cos 2x$
$|a|$ = 1.5, so the amplitude is 1.5
b = 2 so there are 2 cycles from 0 to 2π
$\frac{2\neq}{b} = \frac{2\neq}{2} = \neq$ so the period is π
Divide the period into fourths.
Using the values of the amplitude and period plot the pattern zero-max-zero-min-zero.

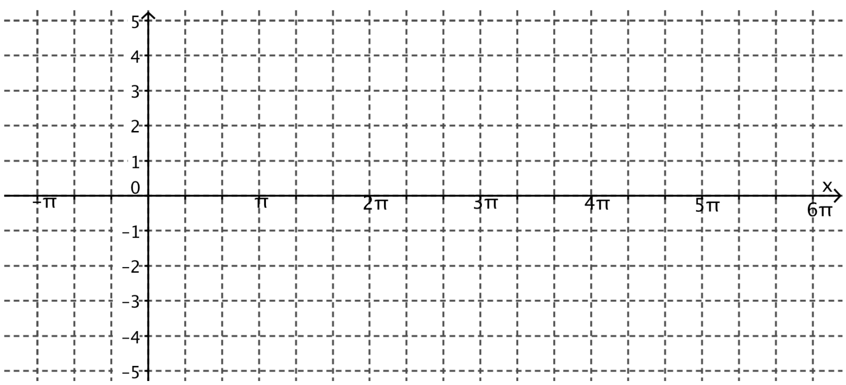


Graphing Sine and Cosine Practice – Amplitude & Period. Graph each function.

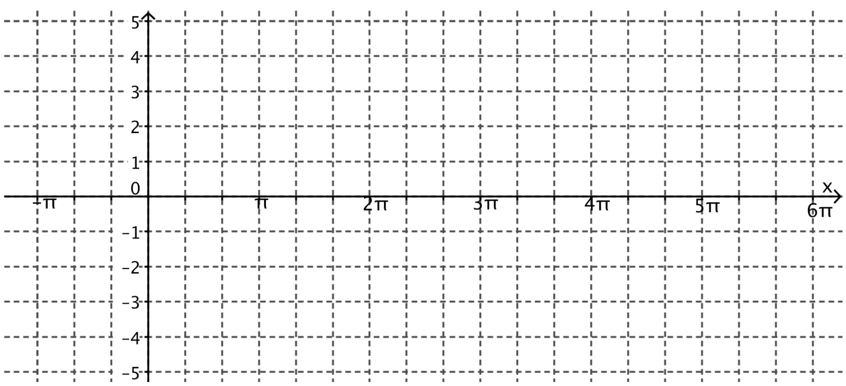
	Amplitude	b – value	period
1. $y = 3\sin x$			
2. $y = -2\cos x$			
3. $y = 0.5 \sin 2x$			
4. $y = 4 \cos (x/2)$			



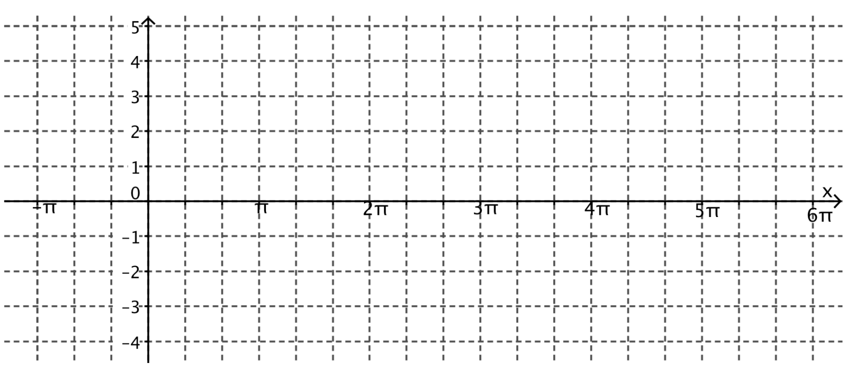
1. $y = 3\sin x$



2. $y = -2\cos x$



3. $y = 0.5 \sin 2x$



4. $y = 4 \cos (x/2)$

Translating Sine and Cosine Functions

For ***y = a sin bx + k*** or ***y = a cos bx + k***

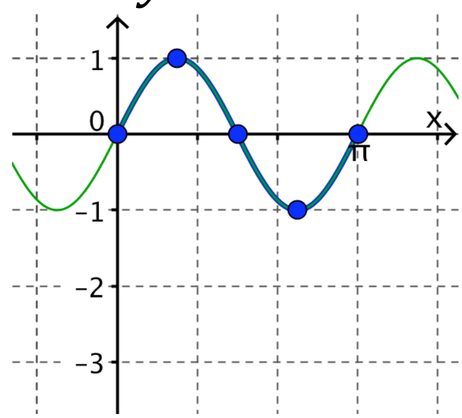
- $|a|$ is the amplitude of the function
- if a is negative the graph flips over the x-axis
- b is the number of cycles in the interval 0 to 2π
- $\frac{2\pi}{b}$ is the period of the function
- k is the vertical shift

Example: Sketch the graph of $y = \sin 2x - \frac{3}{2}$

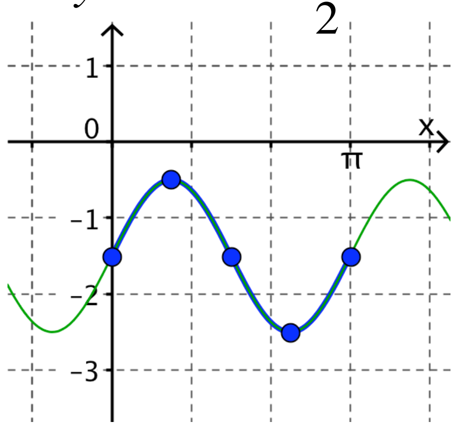
$|a|$ =1, so the amplitude is 1
b = 2 so there are 2 cycles from 0 to 2π
$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ so the period is π
Sketch one cycle of ***y = sin2x***
Use the 5 key points.

Since $k = -\frac{3}{2}$ translate the graph $\frac{3}{2}$ units down. Sketch the final graph.

y = sin2x

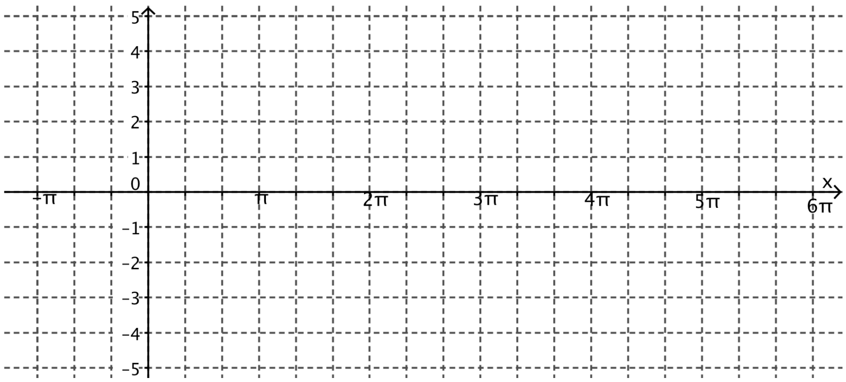


$y = \sin 2x - \frac{3}{2}$

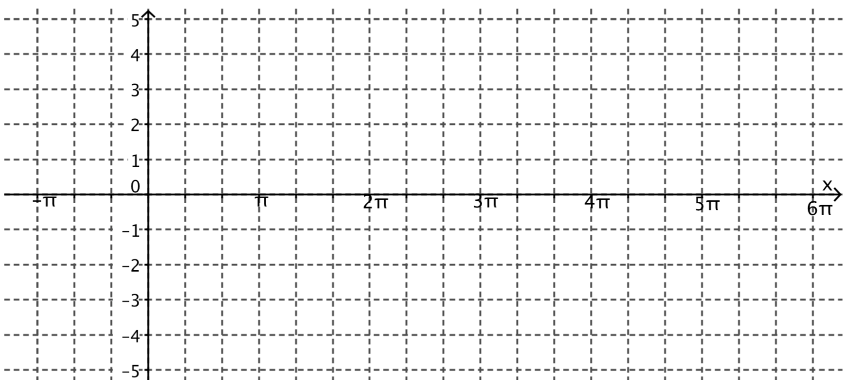


Graphing Sine and Cosine Practice – Amplitude, Period, & Vertical Shift. Graph each function.

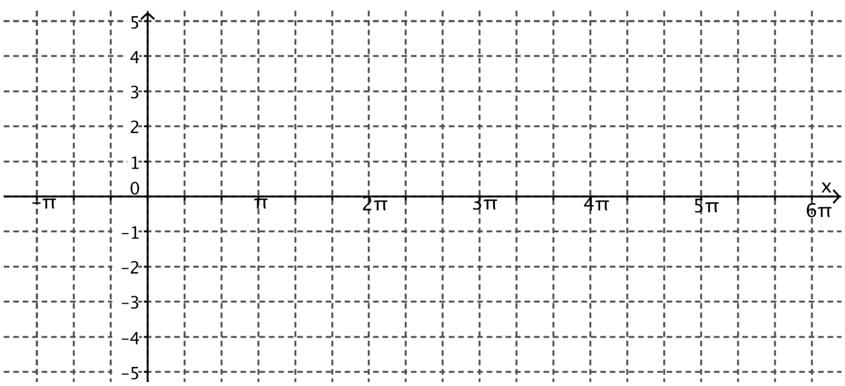
	Amplitude	b – value	period	Vertical Shift
1. $y = -\sin 2x + 3$				
2. $y = 2\sin(x/2) - 1$				
3. $y = 3\cos 3x + 2$				
4. $y = -\cos 2x - 3$				



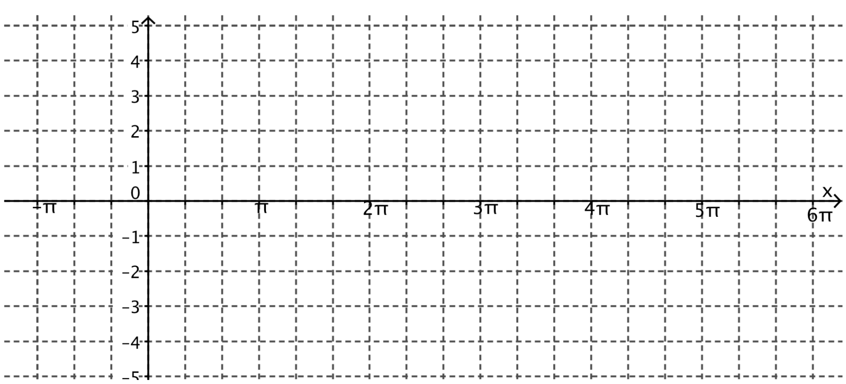
1. $y = -\sin 2x + 3$



2. $y = 2\sin(x/2) - 1$



3. $y = 3\cos 3x + 2$



4. $y = -\cos 2x - 3$

Graphing Worksheet

I. Determine the amplitude, b value, and period for each function.

1. $y = -\frac{1}{2}cosx$
2. $y = 2cos6x$
3. $y = -4sin\frac{\pi}{4}x$
4. $y = 3sin\frac{3}{2}x$
5. $y = -5cos\frac{5\pi}{3}x$
6. $y = cos2x$

Amplitude	b – value	period

II. Find the following and then graph at least 2 cycles of each function on GRAPH paper.

7. $y = 3sinx$
8. $y = 5cosx$
9. $y = 4cos2x$
10. $y = -2sin\frac{1}{2}x$
11. $y = -cos3x$
12. $y = 2sin\frac{1}{3}x$

Amplitude	b-value	period

III. Determine the amplitude, b value, period, and vertical shift for each function.

13. $y = cos2x - 5$
14. $y = 3\cos\frac{\pi}{2} + 4$
15. $y = -\cos 3x - 2$
16. $y = -2\sin \pi x + 1$
17. $y = 4\sin\frac{x}{4} + 2$
18. $y = 3sin6x - 3$

Amplitude	b-value	period	Vertical Shift

IV. Find the following and then graph at least 2 cycles on GRAPH paper.

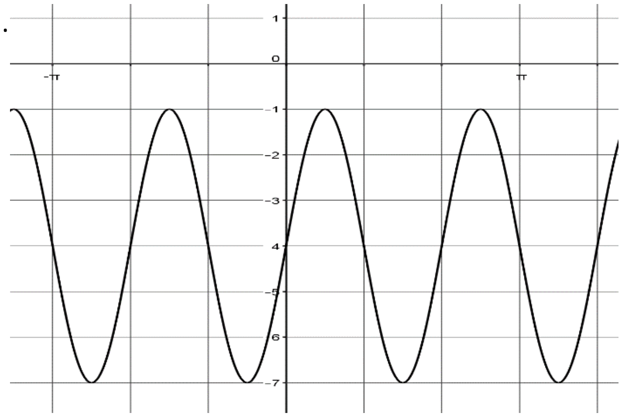
19. $y = cos2x - 1$
21. $y = -\cos x + 2$
22. $y = 2\sin 4x - 3$
23. $y = -3\sin\frac{x}{2} + 1$

Amplitude	b-value	period	Vertical Shift

Writing Equations from Graphs

Write the indicated equation for each graph. **HONORS**

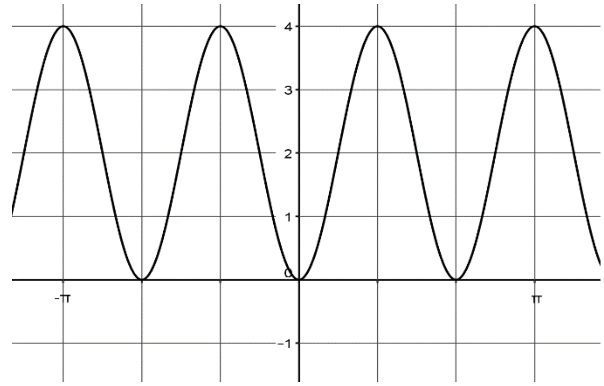
1.



Write a sine equation.

____

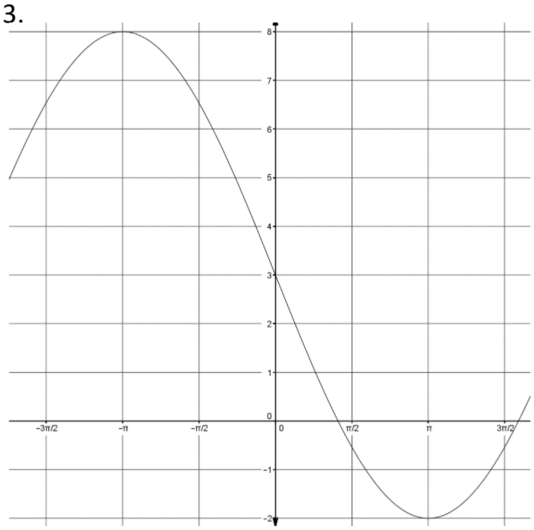
2.



Write a cosine equation.

____

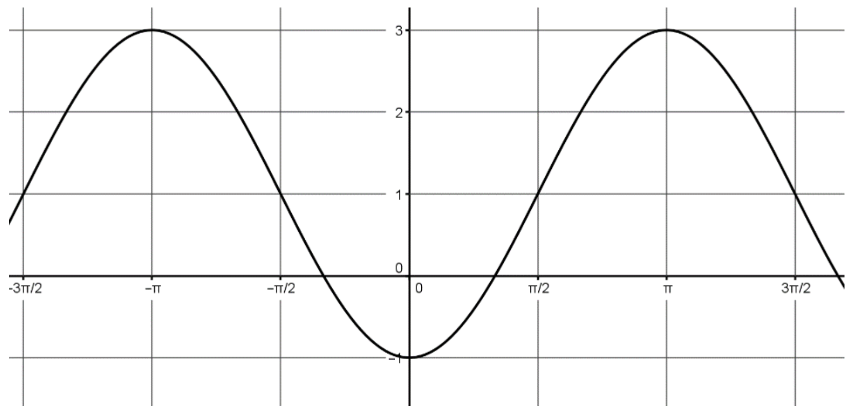
3.



Write a sine equation.

____

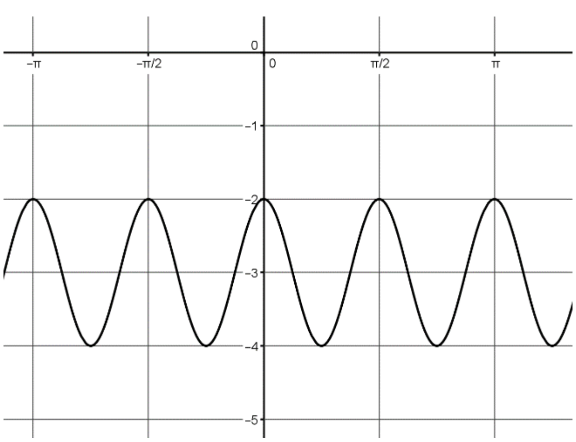
4.



Write a cosine equation.

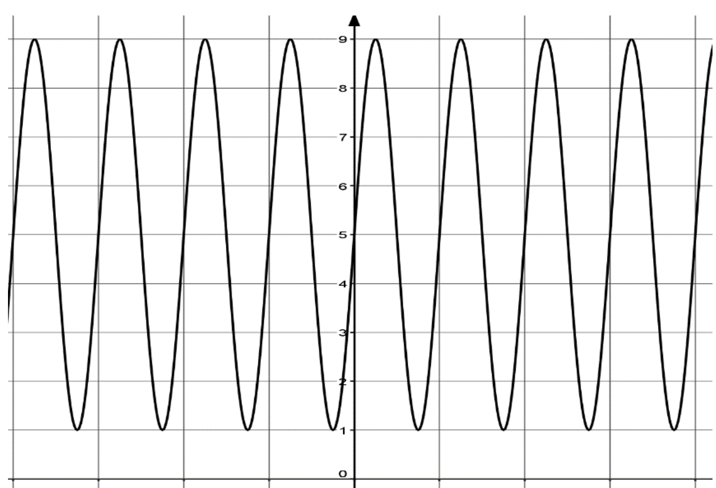
____

5.



Write a cosine equation.

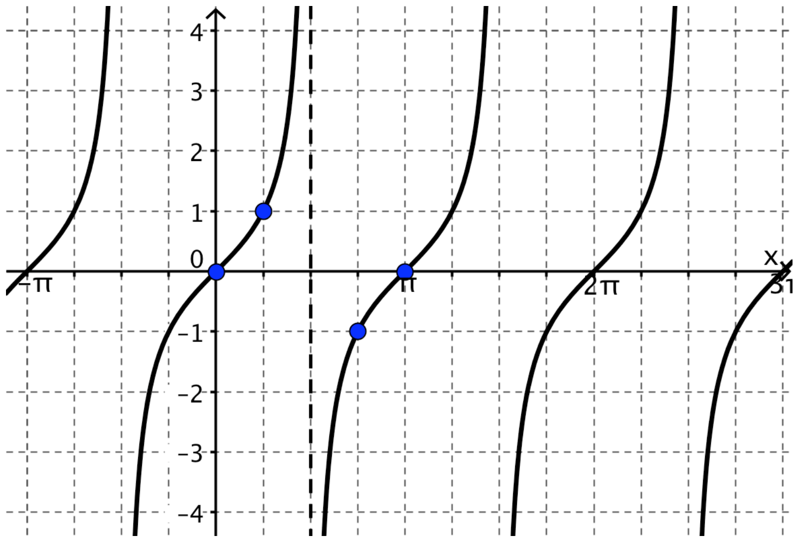
6.



Write a sine equation.

Graphing Tangent **HONORS**

Tangent Function: y = tanx (period = π)



Remember that $\tan x = \dfrac{\sin x}{\cos x}$. Values of x which make the denominator = 0 are not allowed in the domain. The denominator is 0 whenever cos x = 0. Thus the lines $x = \dfrac{\pi}{2} + n\pi$ where n is an integer, are vertical asymptotes of the tangent function. The x-intercepts for the tangent function occur where *tan x = 0,* which means whenever *sin x* = 0.

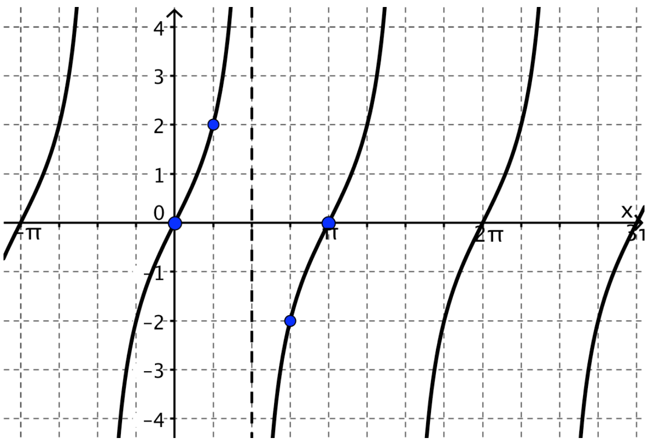
If you think of one cycle as starting on the horizontal at x = 0 and going to x = π, then the vertical asymptote occurs at the half-way point, and at the ¼ and ¾ points the value of y = 1 and – 1 respectively.

For ***y = a tan bx + k*** with ***a≠ 0, b >0*** and **x in radians**:

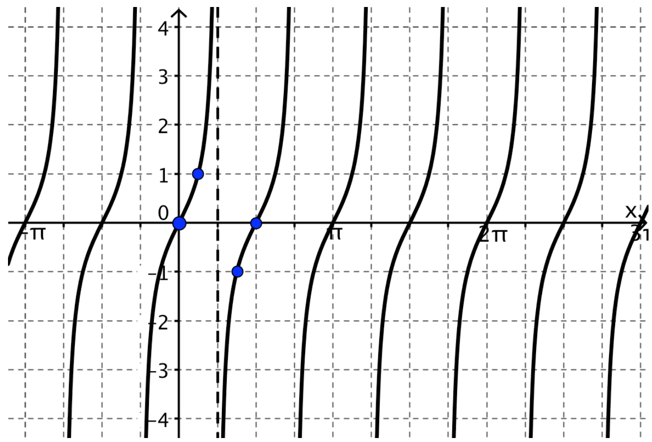
- |*a*| is a vertical stretch/shrink (it changes the value of y at the ¼ and ¾ points)
- if *a* is negative the graph flips over the x-axis
- *b* is the number of cycles in the interval 0 to π
- $\dfrac{\pi}{b}$ is the period of the function
- k is the vertical shift

Tangent Graph Examples:

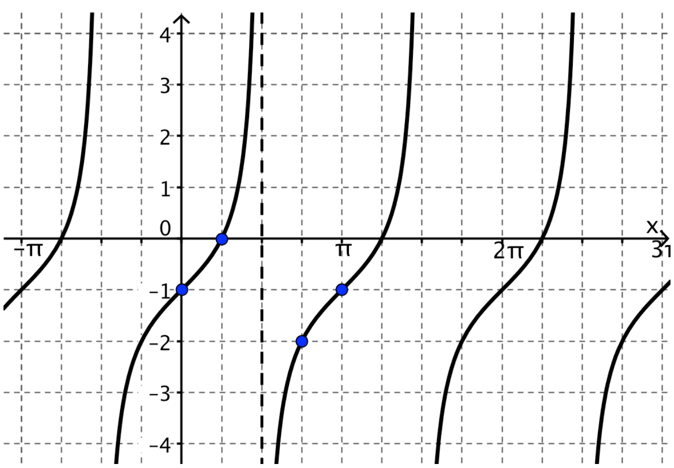
1. $y = 2\tan x$



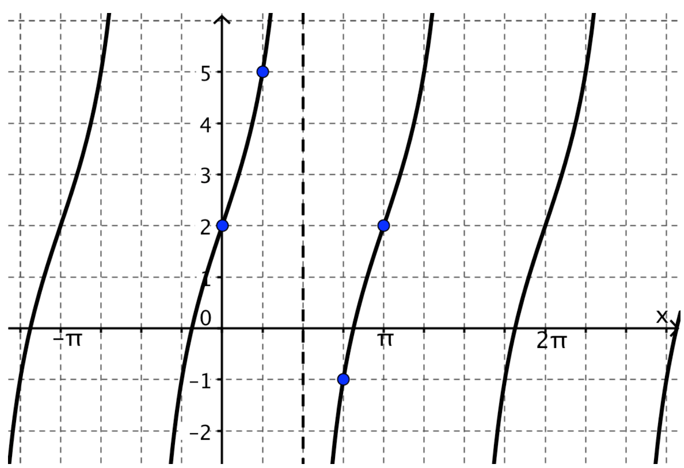
2. $y = \tan 2x$



3. $y = \tan(x) - 1$



4. $y = 3\tan(x) + 2$



Practice: (graph on your own paper)

1. $y = \frac{1}{2}\tan x$
2. $y = \tan\frac{x}{2}$
3. $y = -\tan x + 2$
4. $y = \tan x - 3$
5. $y = 2\tan x - 1$
6. $y = \tan 2x + 3$
7. $y = -\frac{3}{2}\tan\frac{x}{4}$
8. $y = -3\tan 2x + 4$

Word Problems with Sine and Cosine Examples

1. The initial behavior of the vibrations of the note E above middle C can be modeled by $y = 0.5 \sin 660\pi t$.
   a. What is the amplitude of this model?

   b. What is the period of this mode?

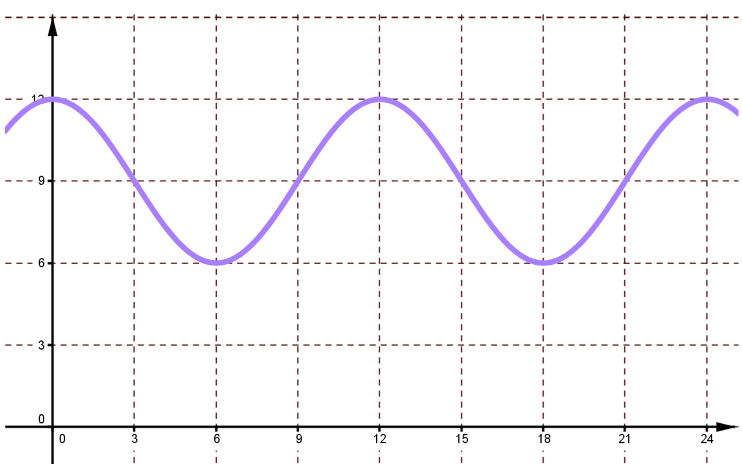
2. A rodeo performer spins a lasso in a circle perpendicular to the ground. The height of the knot from the ground is modeled by $h = -3\cos(\frac{5\pi}{3}t) + 3.5$, where t is the time measured in seconds.
   a. What is the highest point reached by the knot?

   b. What is the lowest point reached by the knot?

   c. What is the period of the model?

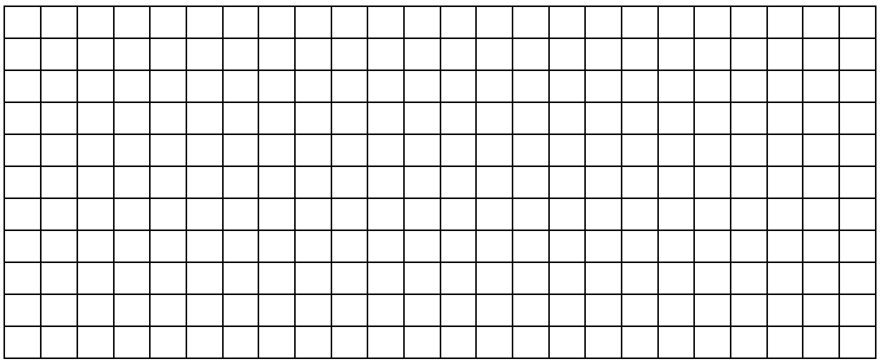
   d. According to the model, find the height of the knot after 25 seconds.

3. The figure shows the depth of water at the end of a boat dock. The depth is 6 feet at low tide and 12 feet at high tide. On a certain day, low tide occurs at 6 A.M. and high tide occurs at noon. If y represents the depth of the water x hours after midnight, use a cosine function of the form $y = A\cos Bx + D$ to model the water's depth.



4. An average seated adult breathes in and out every 4 seconds. The average minimum amount of air in the lungs is 0.08 liter, and the average maximum amount of air in the lunges is 0.82 liter. Suppose the lunges have a minimum amount of air at $t = 0$, where t is the time in seconds.
   a. Write a function that models the amount of air in the lungs.

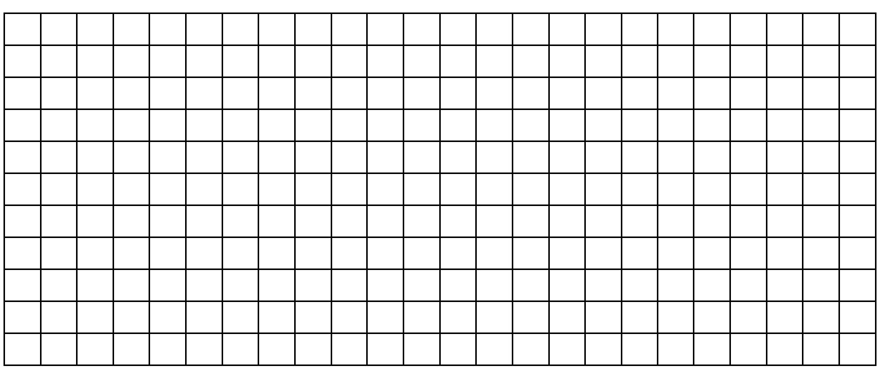
   b. Graph the function.



   c. Determine the amount of air in the lungs at 5.5 seconds.

5. The tide in a coastal city peaks every 11.6 hours. The tide ranges from 3.0 meters to 3.3 meters. Suppose that the low tide is at $t = 0$, where t is the time in hours.
   a. Write a function that models the height of the tide.

   b. Graph the function.



   c. Determine the height of the tide at 6.2 hours.

Word Problems with Sine and Cosine Worksheet

1. If the equilibrium point is $y = 0$, then $y = -4\cos(\frac{\pi}{6}t)$ models a buoy bobbing up and down in the water. Find the period of the function and the location of the buoy at $t = 10$.

2. The function $y = 25\sin(\frac{\pi}{6}t) + 60$, where t is in months and $t = 0$ corresponds to April 15, models the average high temperature in degrees Fahrenheit in Centerville.

   a. Find the period of the function.

   b. What does the period represent?

   c. What is the maximum high temperature?

   d. When does the maximum occur?

3. The population of predators and prey in a closed ecological system tends to vary periodically over time. In a certain system, the population of owls O can be presented by $O = 30\sin\left(\frac{\pi}{10}t\right) + 150$, where t is the time in years since January 1, 2001.

   a. Find the maximum number of owls.

   b. When does the maximum occur?

   c. Find the minimum number of owls

   d. When does the minimum occur?

4. A leaf floats on the water bobbing up and down. The distance between its highest point is 4 centimeters. It moves from its highest point down to its lowest point and back to its highest point every 10 seconds. Write a cosine function the models the movement of the leaf in relationship to the equilibrium point.

5. A person's blood pressure oscillates between 160 and 60. If the heart beats once every second, write a sine function that models this person's blood pressure.

6. Kala is jumping rope, and the rope touches the ground every time she jumps. She jumps at the rate of 40 jumps per minute, and the distance from the ground to the midpoint of the rope at its highest point is 5 feet. At $t = 0$ the height of the midpoint is zero.
   a. Write a function for the height of the midpoint of the rope above the ground after t seconds

   b. Find the height of the midpoint of the rope after 32 seconds.

7. In the wild, predators such as wolves need prey such as sheep to survive. The population of the wolves and the sheep are cyclic in nature. Suppose the population of the wolves W is modeled by $W = 3000 + 1000\sin\left(\frac{\pi t}{6}\right)$ and population of the sheep S is modeled by

   $S = 10{,}000 + 5000\cos\left(\frac{\pi t}{6}\right)$ where t is the time in months.

   a. What is the maximum number and minimum number of wolves?

   b. What the maximum number and minimum number of sheep?

   c. During which months does the wolf population reach a maximum?

   d. During which months does the sheep population reach a maximum?

Test Review

____ 1. Write the standard equation for the circle with center (2, 7), $r = 4$.

a. $(x-7)^2 + (y-2)^2 = 16$ c. $(x-2)^2 + (y-7)^2 = 16$

b. $(x-2)^2 + (y-7)^2 = 4$ d. $(x+2)^2 + (y+7)^2 = 4$

____ 2. Write the standard equation for the circle with center (–6, –8), that passes through (0, 0)

a. $(x-6)^2 + (y-8)^2 = 10$ c. $(x+6)^2 + (y+8)^2 = 14$

b. $(x-6)^2 + (y-8)^2 = 196$ d. $(x+6)^2 + (y+8)^2 = 100$

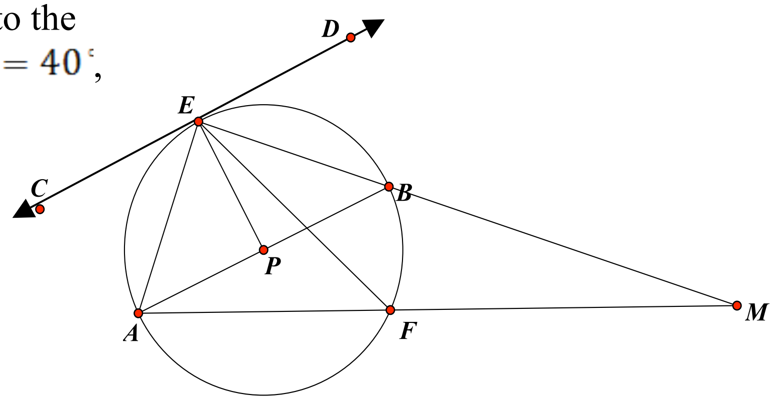
____ 3. Find the center and radius of the circle with equation $(x+9)^2 + (y+5)^2 = 64$.

a. center (5, 9); $r = 8$ c. center (–9, –5); $r = 64$

b. center (9, 5); $r = 64$ d. center (–9, –5); $r = 8$

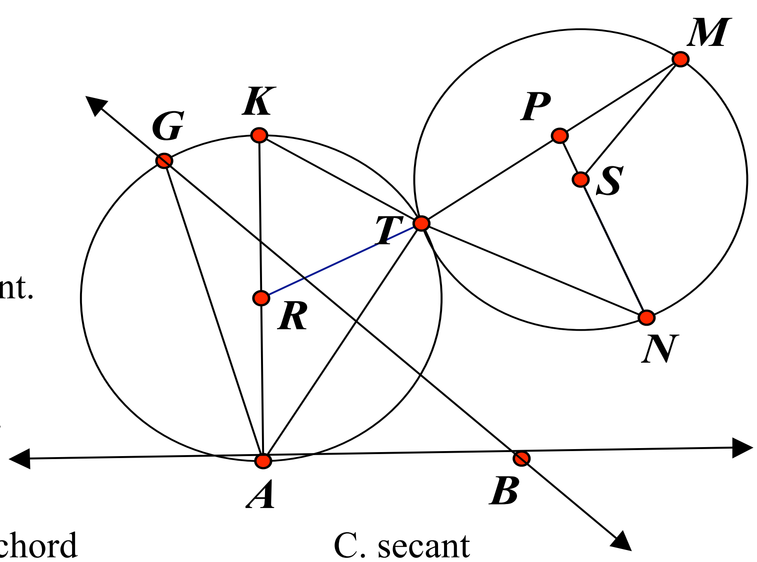
In the figure, $\overline{AB}$ is a diameter, P is the center of the circle, $\overleftrightarrow{CD}$ is a tangent to the circle at E. If $m\widehat{BE} = 100^\circ$ and $m\widehat{BF} = 40^\circ$, find the following measures:

4. $m\widehat{AF}$
5. $m\widehat{AE}$
6. m∠EPB
7. m∠CEA
8. m∠M
9. m∠EAB
10. m∠PEF
11. m∠AEP
12. m∠EFM
13. m∠DEF
14. m∠BAF



Matching. In the figure the two circles, with centers R and S, intersect only at T and $\overline{AB} \perp \overline{RA}$.

15. $\overleftrightarrow{AB}$ is a ______.
16. $\overline{KA}$ is a ______.
17. $\overline{NS}$ is a ______.
18. $\overleftrightarrow{BG}$ is a ______.
19. Circles R and S are ______ tangent.
20. $\overline{KT}$ is a ______.
21. R is a ______.
22. Point P is a(n) ______ of circle S.
23. Point B is a(n) ______ of circle S.



A. diameter B. chord C. secant

D. radius E. center of circle F. tangent

G. interior point H. exterior point I. externally

J. internally

24.In a circle with radius 6, a sector has an area 15π. What is the length of the arc of the sector?

25.The radius of a sector is 12 and the measure of the arc is 130°. Find:

a) the length of the arc   b) the area of the sector

26. If each angle has the given measure and is in standard position, determine the quadrant in which its terminal side lies.

__________ a. $\frac{-5\pi}{6}$ __________ b. 470°

27. Change each degree measure to radian measure in terms of π.

__________ a.. 80° __________ b. 285°

28. Change each radian measure to degrees.

__________ a. $\frac{-\pi}{3}$ __________ b. $\frac{16\pi}{9}$

29. Write the word TRUE or the word FALSE. Determine whether the angles are coterminal.

__________ a. $-215^\circ, 215^\circ$ __________ b. $\frac{-5\pi}{3}, \frac{\pi}{3}$

30. Find the reference angle for each angle with the given measure.

__________ a. 92° __________ b. $\frac{7\pi}{8}$

31. Identify the amplitude, period, phase shift, and vertical shift for each function.

a. $y = -5\cos(3x) + 7$ A:_______ P: _______ Vertical: _____________

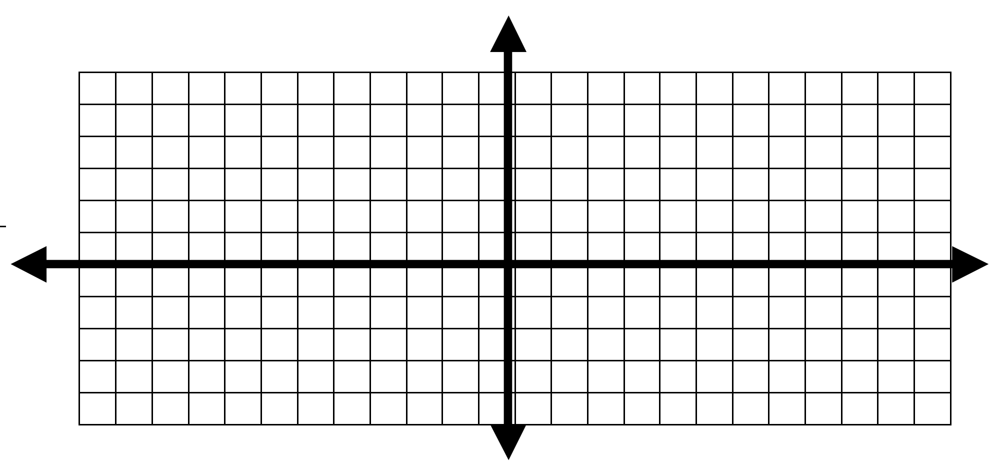
b. $y = 6\sin(4x) - 13$ A:_______ P: _______ Vertical: _____________

32. Graph $y = -3\sin(2x) + 1$

A=_______

Period=_________

Vertical Shift=_____________



33. Find the exact value of each trigonometric function.

a. $\cos\ 30°$ _____ b. $\tan\ 150°$ _____ c. $\sin\ 60°$ _____ d. $\sin\ 225°$ _____

34. Find the following values to the nearest hundredth on the calculator.

__________ a. $\sin 25^{\circ}$ __________ b. $\tan 70^{\circ}$

__________ c. $\cos\frac{6\neq}{5}$ __________ d. $\sin\frac{17\neq}{7}$

35. Find the values of the three given trigonometric functions of an angle in standard position if the point $(-5,8)$ lies on its terminal side.

$\sin\theta =$ __________ $\cos\theta =$ __________ $\tan\theta =$ __________

36. San Francisco Bay has a low tide of 3 ft at midnight and high tide of 15 ft at 5:00 am.

a. Graph the height of tide after midnight.

b. After midnight, when will the tide be low again?

c. After 5:00 am, when will the tide be high again?

d. What is the equation that represents the tide height after midnight?

e. What is the first time after midnight that the height of the water is 11 feet?

Trig Review

Determine the quadrant each angle lies in.

1. $-\frac{7\pi}{4}$ 2. $\frac{8\pi}{3}$ 3. -447° 4. 623° 5. $\frac{23\pi}{6}$

Change each degree to radian measure. (exact value)

6. 220º 7. -442º 8. 124º 9. -450º 10. 15º

Change each radian to degrees.

11. $-\frac{11\pi}{7}$ 12. $\frac{9\pi}{4}$ 13. $-\frac{5\pi}{2}$ 14. $\frac{17\pi}{18}$

Find a positive and negative co-terminal angle for each given angle. Keep units the same.

15. $\frac{17\pi}{6}$ 16. -108º

Find the reference angle for each given angle. Keep the indicated units.

17. 225º 18. 96º 19. -29º 20. 303º

21. $\frac{5\pi}{9}$ 22. $-\frac{18\pi}{7}$ 23. $\frac{23\pi}{5}$ 24. $-\frac{5\pi}{6}$

Find the values of each given the point on the terminal side of .

25. (-2, 7) sinθ = cosθ = tanθ=

26. (1, -9) sinθ = cosθ = tanθ=

27. If θ lies in quadrant II, and sinθ = $\frac{8}{17}$, then tanθ = ___________ and cosθ = _________

28. Find the arc length of a circle given the central angle is 1.5 radians and the radius is 7 cm.

29. An arc has a measure of 6 cm intercepts a central angle of 75º. Find the radius of the circle. Round to the nearest tenths. **(Honors)**

30. Find the area of a sector if its central angle is 35º and the radius of the circle is 12.4 cm. Round to nearest tenths.

Use a calculator and round to FOUR places. Beware of MODE.

31. sin 34º 32. tan(-85.3º) 33. $\cos\left(-\frac{3\pi}{8}\right)$ 34. $\tan\left(\frac{23\pi}{25}\right)$

EXACT VALUES   NO CALC!!!

35. tan 180º 36. cos 315º 37. sin 150º 38. tan 135º 39. sin 0º

40. $\cos\frac{2\pi}{3}$ 41. $\sin\left(-\frac{7\pi}{4}\right)$ 42. $\tan\frac{3\pi}{2}$ 43. $\cos\frac{11\pi}{6}$ 44. sin 3π

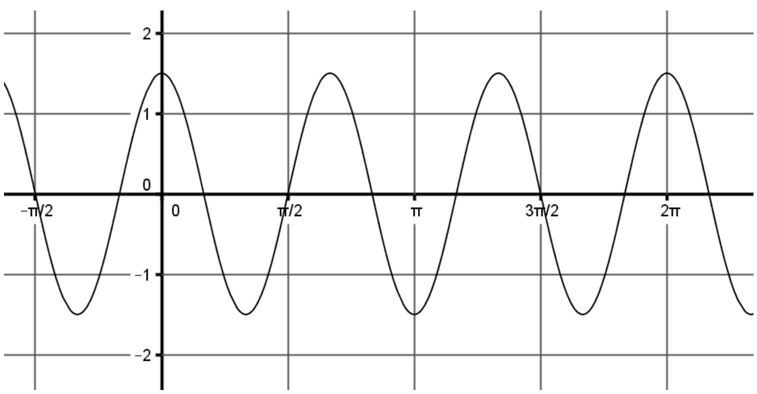
Graph on graph paper

45. $y = 2\sin 2x - 5$ 46. $y = -3\cos(3x + \pi) + 4$ 47. $y = 3\tan 2x - 3$ **(Honors)**

47. Given $h(t) = -32\cos\frac{\pi}{23}t + 48$

____________________= max height __________________= min height ________________= period

48. Write a sine equation for the following graph. **(Honors)**



Honors - Use a calculator and round to FOUR places. Beware of MODE.

49. csc 34º12'26" 50. cot(-85.3º) 51. $\csc\left(-\frac{3\pi}{8}\right)$ 52. $\sec\left(\frac{23\pi}{25}\right)$

Honors - EXACT VALUES   NO CALC!!!

53. sec 180º 54. csc 315º 55. sec 150º 56. cot 135º 57. csc 0º

58. $\sec\left(\frac{2\pi}{3}\right)$ 59. $\cot\left(-\frac{7\pi}{4}\right)$ 60. $\csc\left(\frac{3\pi}{2}\right)$ 61. $\cot\left(\frac{11\pi}{6}\right)$ 62. csc 3π