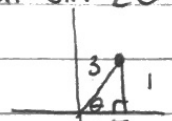
2.3 Double-Angle Formula

1. $\sin\theta = \frac{1}{3}\frac{y}{r}$ θ is in QI

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$

$\boxed{\frac{\sqrt{2}}{9}}$



$3^2 = 1^2 + b^2$

$8 = b^2$

$\sqrt{8} = b$

$2\sqrt{2} = b$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\left(\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2$

$\frac{4\cdot 2}{9} - \frac{1}{9} = \boxed{\frac{7}{9}}$

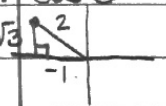
c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\tan\theta = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\frac{2\left(\frac{\sqrt{2}}{4}\right)}{1-\left(\sqrt{2}/4\right)^2} = \frac{\frac{\sqrt{2}}{2}}{1-\frac{2}{16}} = \frac{\frac{\sqrt{2}}{2}}{\frac{16}{16}-\frac{2}{16}}$

$\frac{\sqrt{2}}{2} \div \frac{7}{8} = \frac{\sqrt{2}}{2} \cdot \frac{4}{7}\cdot\frac{8}{7} = \boxed{\frac{4\sqrt{2}}{7}}$

2. $\cos\theta = \frac{-1}{2}\frac{x}{r}$; θ in QII



a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{2}\right) = \boxed{-\frac{\sqrt{3}}{2}}$

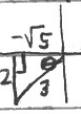
b. $\cos 2\theta = 2\cos^2\theta - 1$

$2\left(\frac{-1}{2}\right)^2 - 1 = \frac{1}{2} - 1 = \boxed{\frac{-1}{2}}$

2. c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2(-\sqrt{3})}{1-(-\sqrt{3})^2} = \frac{-2\sqrt{3}}{4} = \boxed{\frac{-\sqrt{3}}{2}}$

3. $\sin\theta = \frac{-2}{3}\frac{y}{r}$ θ in Q3



$\cos\theta = -\frac{\sqrt{5}}{3}$ $\tan\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-2}{3}\right)\left(\frac{\sqrt{5}}{3}\right) = \boxed{\frac{4\sqrt{5}}{9}}$

b. $\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{-\sqrt{5}}{3}\right)^2 - 1$

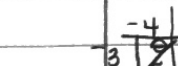
$= \frac{10}{9} - 1 = \boxed{\frac{1}{9}}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{2\sqrt{5}}{5}\right)}{1-\left(\frac{2\sqrt{5}}{5}\right)^2} = \frac{\frac{4\sqrt{5}}{5}}{1-\frac{20}{25}}$

$= \frac{4\sqrt{5}}{5}\cdot\frac{5}{25} = \frac{4\sqrt{5}}{5}\cdot\frac{25}{5} = \boxed{4\sqrt{5}}$

4. $\tan\theta = \frac{3}{4}$ $\pi < \theta < \frac{3\pi}{2}$ QIII



$\sin\theta = \frac{-3}{5}$

$\cos\theta = \frac{-4}{5}$

a. $\sin(2\theta) = 2\sin\theta\cos\theta$

$2\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right) = \boxed{\frac{24}{25}}$

b. $\cos(2\theta) = 1 - 2\sin^2(\theta)$

$1 - 2\left(\frac{-3}{5}\right)^2$

$1 - \frac{18}{25} = \boxed{\frac{7}{25}}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{3/2}{7/16}$

$\frac{3}{2} \cdot \frac{16}{7} = \boxed{\frac{24}{7}}$

c. $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{1}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

d. $\sec^2\theta = \frac{2}{1+\cos 2\theta}$

$= \frac{1}{\cos^2\theta}$

$= \frac{1}{\frac{1+\cos(2\theta)}{2}}$

$= \frac{2}{1+\cos(2\theta)}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\frac{1+\cos(2\theta)}{2} = \cos^2\theta$

5. a. $1 + \sin 2\theta = (\sin\theta + \cos\theta)^2$

$1 + 2\sin\theta\cos\theta$

$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= (\sin\theta + \cos\theta)^2$

b. $\sin 2\theta = 2\cot\theta\sin^2\theta$

$= 2\sin\theta\cos\theta$

$= 2\sin\theta\cos\theta \cdot \frac{\sin\theta}{\sin\theta}$

$= 2\sin^2\theta \cdot \frac{\cos\theta}{\sin\theta}$

$= 2\sin^2\theta\cot\theta$

6. a. $\frac{1-\cos(2\theta)}{2} = \sin^2\theta$

$= \frac{1-(1-2\sin^2\theta)}{2}$

$= \frac{2\sin^2\theta}{2}$

$= \sin^2\theta$ ✓

b. $\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta} = -\sec 2\theta$

$= \frac{1}{\sin^2\theta-\cos^2\theta}$

$= \frac{1}{(1-\cos^2\theta)-\cos^2\theta}$

$= \frac{1}{1-2\cos^2\theta}$

$= \frac{-1}{2\cos^2\theta-1}$

$= \frac{-1}{\cos 2\theta}$

$= -\sec 2\theta$ ✓

c. $\frac{(\sin\theta+\cos\theta)^2}{\sin 2\theta} = \csc 2\theta + 1$

$= \frac{\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{1+2\sin\theta\cos\theta}{2\sin\theta\cos\theta}$

$= \frac{1}{2\sin\theta\cos\theta} + \frac{2\sin\theta\cos\theta}{2\sin\theta\cos\theta}$

$= \frac{1}{\sin 2\theta} + 1$

$= \csc 2\theta + 1$ ✓