Solving Rational Equations

Rational equation:
Equations with a variable in the denominator(s).

Proportion:
One fraction set equal to another.

Extraneous solution:
Answers that are algebraically correct but do not check in the original problem.

Restricted domain:
Instead of all real numbers, the domain has values of x that cannot be used.

Example 1 Solve a proportion

To solve a proportion, cross multiply. Remember to distribute when appropriate.
Then solve for x. Always check for extraneous solutions.

A] $\frac{20}{3x-5} = \frac{5}{x-2}$

$20(x-2) = 5(3x-5)$

$20x-40 = 15x-25$

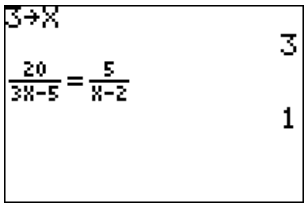
$-15x \quad -15x$

$5x-40 = -25$

$+40 \quad +40$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$ ✓



B] $\frac{x}{x^2-2} = \frac{-1}{x}$

$x^2 = -1(x^2-2)$

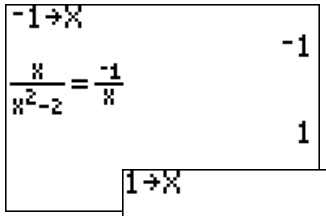
$x^2 = -x^2+2$

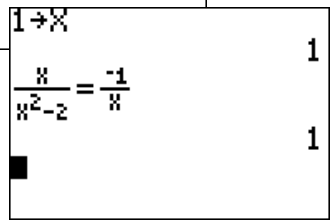
$+x^2 \quad +x^2$

$\frac{2x^2}{2} = \frac{2}{2}$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$ ✓





C] $\frac{x-3}{x+5} = \frac{x}{x+2}$

$x(x+5) = (x-3)(x+2)$

$x^2+5x = x^2+2x-3x-6$

$x^2+5x = x^2-x-6$

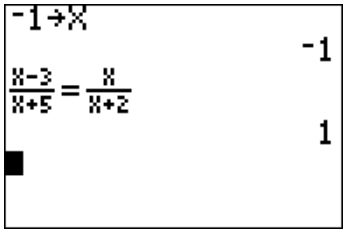
$-x^2 \quad -x^2$

$5x = -x-6$

$+x \quad +x$

$\frac{6x}{6} = \frac{-6}{6}$

$x = -1$ ✓



On your whiteboard...

A] $\dfrac{8}{3x-2}=\dfrac{2}{x-1}$

$8(x-1)=2(3x-2)$

$8x-8=6x-4$

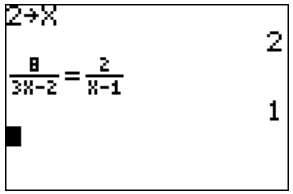
$-6x \quad -6x$

$2x-8=-4$

$+8 \quad +8$

$\frac{2x}{2}=\frac{4}{2}$

$x=2$



B] $\dfrac{x+2}{x-3}=2$

$2(x-3)=1(x+2)$

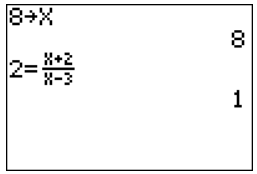
$2x-6=x+2$

$-x \quad -x$

$x-6=2$

$+6 \quad +6$

$x=8$



C] $\dfrac{x-4}{x-1}=\dfrac{10}{x+7}$

$(x-4)(x+7)=10(x-1)$

$x^2+3x-28=10x-10$

$-10x \quad -10x$

$x^2-7x-28=-10$

$+10 \quad +10$

$x^2-7x-18=0$

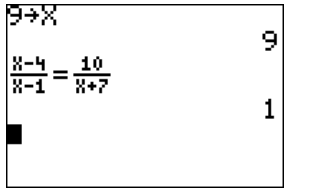
$(x-9)(x+2)=0$

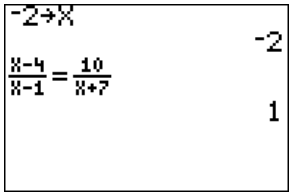
$x=9 \quad x=-2$

-18

$-9 \quad 2$

-7





Example 2 Solve a rational equation

To solve a rational equation, multiply each term by the LCD to eliminate the fractions.
Then solve for x. Remember to check for extraneous solutions.

A] $\frac{2}{3x}+\frac{1}{6}=\frac{4}{3x}$

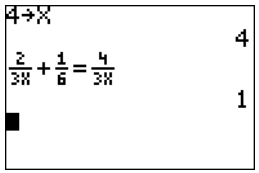
LCD = 6x

$\frac{2}{3x}\cdot\frac{(6x)}{1}+\frac{1}{6}\cdot\frac{(6x)}{1}=\frac{4}{3x}\cdot\frac{(6x)}{1}$

$4+x=8$

$-4 \quad -4$

$x=4$



B] $\frac{1}{x-2}+\frac{2}{1}=\frac{3x}{x+2}$

LCD = (x-2)(x+2)

$\frac{1}{x-2}\cdot\frac{(x+2)(x-2)}{1}+\frac{2}{1}\cdot\frac{(x+2)(x-2)}{1}=\frac{3x}{x+2}\cdot\frac{(x+2)(x-2)}{1}$

$x+2+2(x+2)(x-2)=3x(x-2)$

$x+2+2(x^2-4)=3x^2-6x$

$x+2+2x^2-8=3x^2-6x$

$2x^2+x-6=3x^2-6x$

$-2x^2 \quad -2x^2$

$x-6=x^2-6x$

$-x \quad -x$

$-6=x^2-7x$

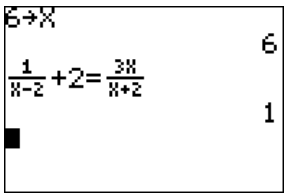
$+6 \quad +6$

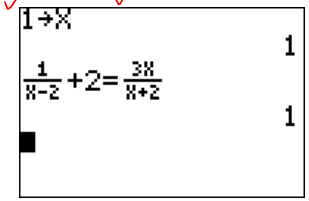
$0=x^2-7x+6$

$0=(x-6)(x-1)$

$x=6 \quad x=1$

6, -6, -1, -7





On your whiteboard...

$$\frac{x}{2x-1}-\frac{2}{2x+1}=\frac{x^2+20}{4x^2-1}$$

$$\frac{x}{2x-1}(2x+1)(2x-1)-\frac{2}{2x+1}(2x+1)(2x-1)=\frac{x^2+20}{4x^2-1}(2x+1)(2x-1)$$

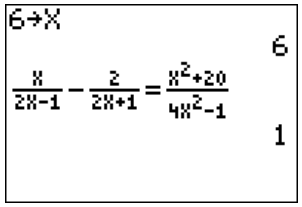
$$x(2x+1)-2(2x-1)=x^2+20$$

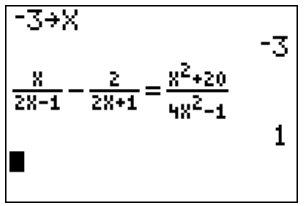
$$2x^2+x-4x+2=x^2+20$$

$$x^2-3x-18=0$$

$$(x-6)(x+3)=0$$

$x=6$ ✓ $x=-3$ ✓





Example 3 Rational equations with extraneous solutions

Extraneous solutions are answers that are algebraically correct but do not check in the original problem. Remember that you can never divide by zero, so any value of x that makes a denominator in the original problem equal zero is restricted from the domain.

A] $\frac{2}{x-3} = \frac{1}{x^2-2x-3}$

$2(x^2-2x-3) = 1(x-3)$

$2x^2-4x-6 = x-3$

$-x \quad -x$

$2x^2-5x-6 = -3$

$+3 \quad +3$

$2x^2-5x-3 = 0$

$-6 \cdot 1 = -6$, $-6+1 = -5$

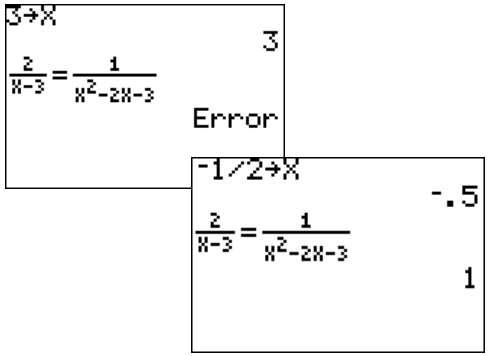
$2x^2$	$-6x$
x	-3

$2x$, 1, x, -3

$(x-3)(2x+1) = 0$

$x = 3$ $\quad x = -\frac{1}{2}$

extraneous



B] $\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$

LCD $= x(x-3)$

$\frac{2}{x-3}(x)(x-3) + \frac{1}{x}(x)(x-3) = \frac{x-1}{x-3}(x)(x-3)$

$2x + x - 3 = x(x-1)$

$3x - 3 = x^2 - x$

$-3x \quad -3x$

$-3 = x^2 - 4x$

$+3 \quad +3$

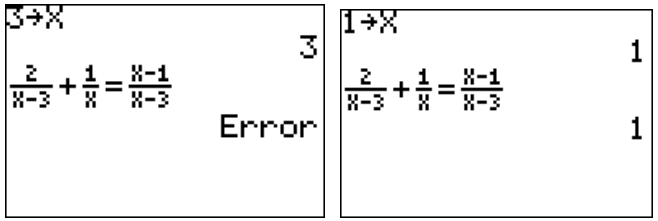
$0 = x^2 - 4x + 3$

$0 = (x-3)(x-1)$

$3 \cdot -1 = -3$, $3-1 = -4$

$x = 3$ $\quad x = 1$

extraneous



On your whiteboard...

$$\frac{6x^2}{x^2-16}-\frac{3x}{x+4}=\frac{4}{x-4}$$

$$\frac{(x+4)(x-4)\,6x^2}{x^2-16}-\frac{3x(x+4)(x-4)}{x+4}=\frac{4\,(x+4)(x-4)}{x-4}$$

$6x^2-3x(x-4)=4(x+4)$

$6x^2-3x^2+12x=4x+16$

$3x^2+8x-16=0$

$(x+4)(3x-4)=0$

$x=-4$ extraneous (causes a 0 in the denominator!)

$3x-4=0$

$x=4/3$ ✓

-48, 12, -4, 8

$3x^2$	$12x$	$3x$
$-4x$	-16	4
-4	4	

x, 4