

# Asymptotes and Holes of Rational Functions

|  | Holes:               | Vertical Asymptotes:                    | Domain:  | Horizontal Asymptotes:             |
|--|----------------------|---|--|------------------------------------|
| 1. $f(x) = \frac{5(x-3)(x+3)}{6(x+3)(x-6)}$  | $x = -3$             | $x = 6$                                 | $(-\infty, -3) \cup$<br>$(-3, 6) \cup$<br>$(6, \infty)$  | $y = \frac{5}{6}$                  |
| 2. $f(x) = \frac{6(x-1)(x+9)(x-8)(x+7)}{(x+9)(x+7)(x-3)}$  | $x = -9$<br>$x = -7$ | $x = 3$                                 | $(-\infty, -9) \cup (-9, -7)$<br>$\cup (-7, 3) \cup (3, \infty)$   | No HA                              |
| 3. $f(x) = \frac{7x(2x-1)(x+4)}{6(2x-1)(2x+5)(3x-4)}$  | $x = \frac{1}{2}$    | $x = \frac{5}{2}$<br>$x = \frac{4}{3}$  | $(-\infty, \frac{1}{2}) \cup$<br>$(\frac{5}{2}, \frac{4}{3}) \cup (\frac{4}{3}, \infty)$<br>$\cup (\frac{4}{3}, \infty)$ | $y = \frac{42}{72} = \frac{7}{12}$ |
| 4. $f(x) = \frac{x^3 + 8}{x^3 - 4x^2 - 12x + 16}$<br><small><math>\frac{(x+2)(x^2-2x+4)}{x^2(3x^2-4x-4)}</math><br/>Done in class!</small> | $x = -2$             | $x = 2$<br>$x = 0$<br>$x = \frac{4}{3}$ | $(-\infty, -2) \cup$<br>$(-2, 0) \cup (0, \frac{4}{3})$<br>$\cup (\frac{4}{3}, 2) \cup (2, \infty)$                      | $y = 0$                            |
| 5. $f(x) = \frac{4x+16}{x-4} = \frac{-4(x-4)}{x-4}$  | $x = 4$              | none                                    | $(-\infty, 4) \cup (4, \infty)$  | $y = -4$                           |
| 6. $f(x) = \frac{x^3 + 2x^2 - 24x}{x^2 + 28x - 147}$<br><small><math>\frac{x(x^2+2x-24)}{x(x+4)(x-7)}</math></small>                       | $x = 3$              | $x = -7$                                | $(-\infty, -7) \cup (-7, 3)$<br>$\cup (3, \infty)$   | None                               |
| 7. $f(x) = \frac{x+3}{2x+6} = \frac{x+3}{2(x+3)}$  | none                 | $x = -3$                                | $(-\infty, -3) \cup$<br>$(-3, \infty)$   | $y = \frac{1}{2}$                  |
| 8. $f(x) = \frac{x+4}{x-4} = \frac{x-4}{(x-2)(x+2)}$   | none                 | $x = 2$<br>$x = -2$                     | $(-\infty, -2) \cup$<br>$(-2, 2) \cup$<br>$(2, \infty)$  | $y = 0$                            |
| 9. $f(x) = \frac{x^2 - 9x}{x^2 - 6x - 9}$<br><small><math>\frac{x(x-9)}{3(x-3)(x+3)}</math></small>  | $x = 3$              | $x = -1$                                | $(-\infty, -1) \cup$<br>$(-1, 3) \cup (3, \infty)$   | None                               |
| 10. $f(x) = \frac{3x-12}{5x^2-10x-40}$<br><small><math>\frac{3(x-4)}{5(x^2-2x-8)}</math></small>   | $x = 4$              | $x = -2$                                | $(-\infty, -2) \cup$<br>$(-2, 4) \cup$<br>$(4, \infty)$  | $y = 0$                            |
| 11. $f(x) = \frac{x^3 + 2x^2 - 3x}{x^3 + x^2 - 25x - 25}$<br><small><math>\frac{x(x+2)(x-1)}{x^2(x+1)-25(x+1)}</math></small>              | None                 | $x = -5$<br>$x = 5$<br>$x = -1$         | $(-\infty, -5) \cup$<br>$(-5, -1) \cup (-1, 5)$<br>$\cup (5, \infty)$  | $y = 1$                            |
| 12. $f(x) = \frac{x^3 - 256}{x^2 + 7x + 12}$<br><small><math>\frac{(x+4)(x+4)(x-4)}{(x+4)(x+3)}</math></small>                             | $x = -4$             | $x = 3$                                 | $(-\infty, -4) \cup$<br>$(-4, 3) \cup (3, \infty)$   | None                               |