Solving Exponential and Logarithmic Equations

1. Solve each exponential equation.

a. $6^{x-2} = 6^{3x-4}$

x-2 = 3x – 4
2 = 2x
x = 1

b. $8^{x-1} = \left(\frac{1}{4}\right)^{1-x}$

$(2^3)^{x-1} = (2^{-2})^{1-x}$
3x – 3 = -2 + 2x
x = 1

c. $5^{x-2} = \frac{1}{125^x}$

$5^{x-2} = 5^{-3x}$
x – 2 = -3x
x = 1/2

d. $4^{5-x} = 128$

$(2^2)^{5-x} = 2^7$
10 – 2x = 7
x = 3/2 = 1.5

e. $3^{x-1} = \left(\sqrt{3}\right)^{x+1}$

x – 1 = 1/2x + ½
2x – 2 = x + 1
x = 3

f. $125^{x-1} = \left(\frac{1}{5}\right)^{1-2x}$

3x – 3 = -1 + 2x
x = 2

g. $2^{3x-1} = 4^{x+2}$

3x – 1 = 2x + 4
x = 5

h. $7^{2x+4} = \left(\frac{1}{49}\right)^{x-3}$

2x + 4 = -2x + 6
4x = 2
x = 1/2

i. $8^{2x-2} = 4^{2-x}$

6x – 6 = 4 – 2x
8x = 10
x = 5/4 = 1.25

j. $10^x = 3.91$

$\log 10^x = \log 3.91$

x = log 3.91
X = .5922

k. $e^x = 5.7$

$\ln e^x = \ln 5.7$

x = ln 5.7
X = 1.7405

l. $5^x = 17$

$\log 5^x = \log 17$

$\mathbf{x} = \frac{\log 17}{\log 5} = 1.7604$

m. $5e^x = 23$

$e^x = 23/5$

$\ln e^x = \ln 4.6$

x = ln 4.6
X = 1.5261

n. $e^{1-5x} = 793$

$\ln e^{1-5x} = \ln 793$

1-5x = ln 793

$\mathbf{x} = \frac{\ln 793 - 1}{-5}$
X = -1.1352

o. $e^{5x-3} - 2 = 10{,}476$

$\ln e^{5x-3} = 10{,}478$

5x – 3 = ln 10,478

$\mathbf{x} = \frac{\ln 10{,}478 + 3}{5}$
X = 2.4514

p. $7^{x+2} = 410$

$\log 7^{x+2} = \log 410$

(x + 2)log 7 = log 410

xlog 7 + 2log 7 = log 410

$\mathbf{x} = \frac{\log 410 - 2\log 7}{\log 7}$

$X = \frac{\log 410}{\log 7} - 2 = 1.0917$

q. $3^{\frac{x}{7}} = .2$

$\log 3^{\frac{x}{7}} = \log .2$

xlog3 = 7log .2

$\mathbf{x} = \frac{7\log .2}{\log 3}$
X = -10.2548

r. $e^{4x} - 5e^{2x} - 24 = 0$

$(e^{2x} - 8)(e^{2x} + 3)$

$e^{2x} = 8 \quad e^{2x} = -3$

$\mathbf{x} = \frac{\ln 8}{2}$ ~~$\mathbf{x} = \frac{\ln -3}{2}$~~
X = 1.0397

s. $e^{2x} - 2e^{x} - 3 = 0$

$(e^{x} + 1)(e^{x} - 3) = 0$

$e^{x} = -1 \qquad e^{x} = 3$

$\mathbf{x = \cancel{\ln -1}} \quad \mathbf{x = \ln 3}$

$X = 1.0986$

t. $e^{4x} + 5e^{2x} - 24 = 0$

$(e^{2x} + 8)(e^{2x} - 3) = 0$

$e^{2x} = -8 \qquad e^{2x} = 3$

$\mathbf{x = \cancel{\frac{\ln -8}{2}}} \quad \mathbf{x = \frac{\ln 3}{2}}$

$X = .5493$

u. $e^{4x} - 3e^{2x} - 18 = 0$

$(e^{2x} + 3)(e^{2x} - 6)$

$e^{2x} = -3 \qquad e^{2x} = 6$

$\mathbf{x = \cancel{\frac{\ln -3}{2}}} \quad \mathbf{x = \frac{\ln 6}{2}}$

$X = .8959$

v. $3^{2x} + 3^{x} - 2 = 0$

$(3^{x} - 1)(\cancel{3^{x} + 2}) = 0$

$\mathbf{x = \frac{\log 1}{\log 3} = 0}$

w. $2^{2x} + 2^{x} - 12 = 0$

$(\cancel{2^{x} + 4})(2^{x} - 3) = 0$

$\mathbf{x = \frac{\log 3}{\log 2}}$