Name: ___________________________

Pre-Calculus – Unit 5 – Exponentials and Logarithms

5.1 Basics of Exponentials and Logarithms

Write in exponential form.

1. $6 = \log_2 64$
2. $2 = \log_9 x$
3. $3 = \log_b 27$
4. $\log_5 125 = y$

Write in logarithmic form.

5. $5^4 = 625$
6. $5^{-3} = \frac{1}{125}$
7. $\sqrt[3]{64} = 4$
8. $15^2 = x$
9. $b^3 = 343$
10. $8^y = 300$

Evaluate.

11. $\log_7 49$
12. $\log_3 27$
13. $\log_6 \sqrt{6}$
14. $\log_3 \frac{1}{9}$
15. $\log_{81} 9$
16. $\log_{11} 11$
17. $\log_6 1$
18. $\log_4 4^5$
19. $7^{\log_7 23}$
20. $\log 1000$
21. $\log 10^8$
22. $10^{\log 53}$
23. $\log_6 17$
24. $\log_{16} 57.2$
25. $\log_{0.3} 19$
26. $\log_\pi 400$

Solve for the variable.

27. $\log_7(x + 2) = -2$
28. $\log_5 x = 3$

5.2 Properties of Exponentials and Logarithms

Expand.

1. $\log_8(13 \cdot 7)$
2. $\log_9(9x)$
3. $\log(10000x)$
4. $\log_9\left(\frac{9}{x}\right)$
5. $\log\left(\frac{x}{100}\right)$
6. $\log_5\left(\frac{125}{x}\right)$
7. $\ln\left(\frac{e^4}{8}\right)$
8. $\log_b x^7$
9. $\log M^{-8}$
10. $\ln \sqrt[7]{x}$
11. $\log_b(xy^3)$
12. $\log_5\left(\frac{\sqrt{x}}{25}\right)$
13. $\log_8\left(\frac{64}{\sqrt{x+1}}\right)$
14. $\log_b\left(\frac{x^3 y}{z^2}\right)$
15. $\ln \sqrt{ex}$
16. $\log_5 \sqrt{\frac{x}{y}}$
17. $\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$
18. $\log_2 \sqrt[5]{\frac{xy^4}{16}}$
19. $\ln\left[\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right]$
20. $\log\left[\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right]$

Condense.

21. $\log 250 + \log 4$
22. $\ln x + \ln 3$
23. $\log_3 405 - \log_3 5$
24. $\log(3x+7) - \log x$
25. $\log x + 7\log y$
26. $\frac{1}{3}\ln x + \ln y$
27. $5\log_b x + 6\log_b y$
28. $7\ln x - 3\ln y$
29. $2\ln x - \frac{1}{2}\ln y$
30. $8\ln(x+9) - 4\ln x$
31. $4\ln x + 7\ln y - 3\ln z$
32. $\frac{1}{3}(\log_4 x - \log_4 y)$
33. $\frac{1}{3}(\log_4 x - \log_4 y) + 2\log_4(x+1)$
34. $\frac{1}{3}[5\ln(x+6) - \ln x - \ln(x^2-25)]$
35. $\log x + \log 15 + \log(x^2-4) - \log(x+2)$

Evaluate to 4 decimal places.

36. $\log_6 17$
37. $\log_{16} 57.2$
38. $\log_{0.3} 19$
39. $\log_{\pi} 400$

5.3 Solving Exponential and Logarithmic Equations

Solve each exponential equation.

1. $6^{x-2} = 6^{3x-4}$
2. $8^{x-1} = \left(\frac{1}{4}\right)^{1-x}$
3. $5^{x-2} = \frac{1}{125^x}$
4. $4^{5-x} = 128$
5. $3^{x-1} = \left(\sqrt{3}\right)^{x+1}$
6. $125^{x-1} = \left(\frac{1}{5}\right)^{1-2x}$
7. $2^{3x-1} = 4^{x+2}$
8. $7^{2x+4} = \left(\frac{1}{49}\right)^{x-3}$
9. $8^{2x-2} = 4^{2-x}$
10. $10^x = 3.91$
11. $e^x = 5.7$
12. $5^x = 17$
13. $5e^x = 23$
14. $e^{1-5x} = 793$
15. $e^{5x-3} - 2 = 10476$
16. $7^{x+2} = 410$
17. $3^{\frac{x}{7}} = 0.2$
18. $e^{4x} - 5e^{2x} - 24 = 0$
19. $e^{2x} - 2e^x - 3 = 0$
20. $e^{4x} + 5e^{2x} - 24 = 0$
21. $e^{4x} - 3e^{2x} - 18 = 0$
22. $3^{2x} + 3^x - 2 = 0$
23. $2^{2x} + 2^x - 12 = 0$

5.4 Applications of Exponentials and Logarithms.

1. Suppose that the value of a computer depreciates at a rate of 25% per year. Determine the value of a laptop computer two years after it was purchased for $3,750.

2. Mexico has a population of about 100 million people, and it is estimated that the population will double in 21 years. If population growth continues at the same rate, estimate the population I 30 years?

3. A researcher estimates the initial population of honeybees in a colony to be 500. If the bees are increasing at a rate of 14% per week, what is the expected population in 22 weeks?

4. If a farmer uses 25 pounds of insecticide, assuming its half-life is 12 years, how many pounds will still be active after 5 years? After 20 years?

5. Carol won $5,000 in a raffle. She would like to invest her winnings in a money market account that provides an APR of 6% compounded quarterly. Does she have to invest all of it in order to have $9,000 at the end of 10 years? Show work to explain your answer.

6. Mike would like to have $20,000 cash for a new car 5 years from now. How much should be placed in an account now if that account pays 9.75% compounded weekly?

7. If you invest $7,500 in an account paying 8.35% compounded continuously, how much money will be in the account at the end of 12 years?

8. Compare the balance after 30 years of a $15,000 investment earning 12% interest compounded continuously to the same investment compounded quarterly.

9. Ana is trying to save for a new house. How many years, to the nearest year, will it take Ana to triple the money in her account if it is invested at 7% compounded annually?

10. At what annual percentage rate (to the nearest hundredth of a percent) compounded continuously will $6,000 have to be invested to amount to $11,000 in 8 years?

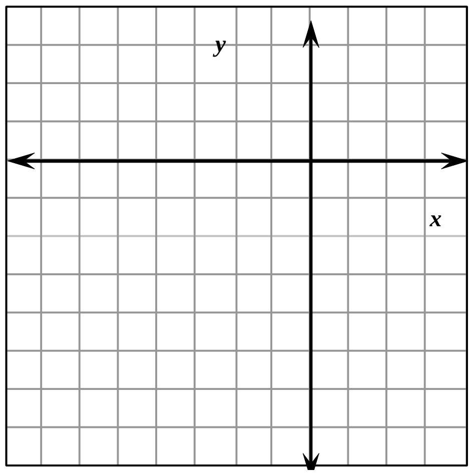
11. How many years, to the nearest year, will it take for the balance of an account to double if it is gaining 6% interest compounded semi-annually?

12. If the world population is about 6 billion people now and if the population grows continuously at an annual rate of 1.7%, what will the population be (to the nearest billion) in 10 years from now?

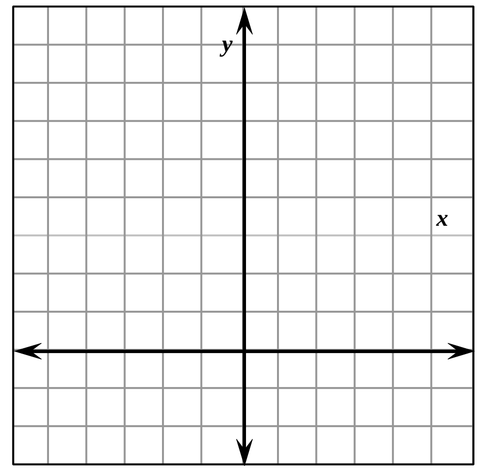
5.5 Exponentials and Logarithms Review

Graph the following functions by showing a series of transformations.

1. $f(x) = -2^{x+3}$



2. $f(x) = 2\log_3(x+2) - 1$



Express as a logarithm.

3. $6^{-3} = \frac{1}{216}$
4. $27^{\frac{4}{3}} = 81$

Express as an exponent.

5. $\log_b x = w$
6. $\log_4 \frac{1}{256} = -4$

Find each logarithm.

7. $\log_3 27$
8. $\log_9 81$
9. $\log_{\frac{1}{3}} 81$
10. $\log_{\frac{1}{2}} \frac{1}{16}$

Solve for x.

11. $\log_x 16 = 4$
12. $\log_8 4 = x$
13. $\log_8 x = -\frac{4}{3}$
14. $\log_{\sqrt{5}} x = 4$
15. $\log_{25} 125 = x$
16. $\log_x 27 = \frac{3}{4}$
17. $\log_2(3x - 4) = 3$
18. $\ln x = 2$

Solve each equation.

19. $\log_8(3x + 7) = \log_8(7x + 4)$
20. $\log_4(2x - 1) = \log_4 16$
21. $\log_{10}\sqrt{10} = x$
22. $\log_7(8x + 20) = \log_7(x + 6)$
23. $\log_{12}(x - 9) = \log_{12}(3x - 13)$
24. $\log_5(x^2 - 30) = \log_5 6$

Write as a single logarithm.

25. $\log_2 a + \log_2 b + \log_2 c$
26. $3\log_b 2a$
27. $2\log_5 x - 3\log_5 y$
28. $(2\log_x 3 + \log_x 6) - \log_x 2y$

Write in expanded form.

29. $\log_5 (ab)^3$

30. $\log_6 \frac{\sqrt{a}}{b}$

Solve for x.

31. $\log_5 x = 2\log_5 10$

32. $\log x = \log 10 - \log 5$

33. $\log x = \frac{1}{2}\log 81 - \frac{1}{3}\log 27$

34. $2\log_5 x = \log_5 12 + \log_5 75$

35. $\log_7 x = 4\log_7 2 + (\log_7 3 - \log_7 6)$

36. $\log 3x = \log 12 + 2(\log 5 - \log 2)$

37. $\log_3 x + \log_3 (x - 8) = 2$

38. $\log_2 (x + 3) = \log_2 (x - 3) = 4$

Simplify.

39. $\log_2 (\log_2 (\log_2 16))$

40. $\log_3 (\log_3 (\log_3 27))$

41. $\log_{36} 6 \cdot \log_{36} 6$

42. $10^{\log_{10} 12 - \log_{10} 2}$

43. $8^{3\log_8 3 - \log_8 5}$

44. $e^{\ln 4x} = \ln 9.4$

45. $\ln e^{1.32x} = 5.8$

46. $x = \log_8 84.3$

47. $2500 = 4e^{0.58x}$

48. $\ln x = -6.5$

49. $\frac{1}{3^x} = 12$

50. If \$750 is invested at 8% annual interest that is compounded monthly when will the investment be worth \$1600?

51. John's new house in Apex is valued at \$105,000. The area he lives in has had a steady rate of appreciation for homes of 12% per year. At this steady rate, when will his house be worth ½ million dollars?

52. If \$50 is invested at 8% annual interest that is compounded continuously when will the investment be worth \$200?

53. A certain bacteria can grow from 40 to 185 in 3.5 hours. Find the constant k for the bacteria.

54. A piece of office equipment worth \$8500 depreciates at 9% per year for the first ten years. At this rate when will the piece of equipment be worth \$5000?

55. A radioactive element has a half-life of 10 hours. If you have 300 g of the element initially, how much remains after 25 hours?