

## 4.4 Sequences and Series Review

1. Classify each sequence as arithmetic, geometric, or neither. If arithmetic or geometric, find the common difference/ratio.

a.  $-2, 6, -18, 54, \dots$  geometric  
 $r = -3$

b.  $3, 6, -12, -24, \dots$  neither

c.  $1, -1, -3, -5, \dots$  arithmetic  
 $d = -2$

d.  $9, 3, 1, \frac{1}{3}, \dots$   
 geometric  $r = \frac{1}{3}$

2. Write a recursive definition for each sequence.

a.  $8, 5, 2, -1, \dots$   $a_1 = 8$   
 $a_n = a_{n-1} - 3$

b.  $12, 4, \frac{4}{3}, \frac{4}{9}, \dots$   $a_1 = 12$   
 $a_n = \frac{1}{3} a_{n-1}$

c.  $-10, 20, -40, 80, \dots$   
 $a_1 = -10$   $a_n = -2a_{n-1}$

d.  $-1, 0, 1, 2, \dots$   
 $a_1 = -1$   $a_n = a_{n-1} + 1$

3. Given the recursive definition, find the fourth term of each sequence.

a.  $a_1 = 4; a_n = 19a_{n-1}$   
 $a_2 = 19(4) = 76$   $a_3 = 19(76) = 1444$   $a_4 = 19(1444) = 27436$

b.  $a_1 = 15; a_n = a_{n-1} + 3$   
 $a_1 = 15$   $a_2 = 15 + 3 = 18$   $a_3 = 18 + 3 = 21$   $a_4 = 21 + 3 = 24$

c.  $a_1 = -2; a_n = -3a_{n-1}$   
 $a_1 = -2$   $a_2 = -3(-2) = 6$   $a_3 = -3(6) = -18$   $a_4 = -3(-18) = 54$

d.  $a_1 = 5; a_n = a_{n-1} - 17$   
 $a_1 = 5$   $a_2 = 5 - 17 = -12$   $a_3 = -12 - 17 = -29$   $a_4 = -29 - 17 = -46$

4. Write the explicit formula for each sequence.

a.  $7, 3, -1, -5, \dots$   
 $a = 7$   $d = -4$   $a_n = 7 + (n-1)(-4)$

b.  $99, 33, 11, \frac{11}{3}, \dots$   
 $a_1 = 99$   $r = \frac{1}{3}$   $a_n = 99 \cdot \left(\frac{1}{3}\right)^{n-1}$

c.  $-37, -29, -21, -13, \dots$   
 $a = -37$   $d = 8$   $a_n = -37 + (n-1)(8)$

d.  $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \dots$   
 $a = \frac{1}{3}$   $r = \frac{2}{3}$   $a_n = \frac{1}{3} \left(\frac{2}{3}\right)^{n-1}$

5. Find the 10th term of each sequence.

a.  $4, 8, 16, 32, \dots$   
 $a_1 = 4$   $r = 2$   $a_{10} = 4 \cdot 2^9 = 2048$

b.  $1, 4, 7, 10, \dots$   
 $a_1 = 1$   $d = 3$   $a_{10} = 1 + (9)(3) = 28$

c.  $\frac{1}{5}, -1, 5, -25, \dots$   
 $a_1 = \frac{1}{5}$   $r = -5$   $a_{10} = \frac{1}{5} (-5)^9 = -390625$

d.  $-8, -10, -12, -14, \dots$   
 $a_1 = -8$   $d = -2$   $a_{10} = -8 + (9)(-2) = -26$

6. Find the indicated sum.

a. Find the sum of the series  
 $40 + 20 + 10 + 5, \dots$   $S = \frac{40}{1 - \frac{1}{2}} = 80$   
 $a_1 = 40$   $r = \frac{1}{2}$  *converges geometric infinite*

b. Find the sum of the first 10 terms of the series  $18 + 38 + 58 + 78, \dots$   
 $a_1 = 18$   $d = 20$   $S_{10} = \frac{10}{2} (18 + 198) = 1080$   
 $a_{10} = 18 + (9)(20) = 198$

- c. Find the sum of the first 8 terms of the series  $3 - 7 - 7 - 27, \dots$   
 omit

d. Find the sum of the first 9 terms of the series  $-1 + 2 - 4 + 8, \dots$   
 $a_1 = -1$   $r = -2$   $S_9 = \frac{-1(1 - (-2)^9)}{1 - (-2)} = 171$

e. Find the sum of the series

$$\frac{1}{36} + \frac{1}{6} + 1 + 6, \dots$$

$a_1 = \frac{1}{36}$     divergent  
 $r = 6$         no sum

f. Find the sum of the first 8 terms of

the series  $18 + 12 + 8 + 16\frac{2}{3}, \dots$

$$a_1 = 18 \quad S_8 = \frac{18(1 - (\frac{2}{3})^8)}{1 - \frac{2}{3}} = \boxed{51.893}$$

$r = \frac{2}{3}$

g. Find the sum of the series

$60 - 15 + 15\frac{1}{4} - 15\frac{1}{16}, \dots$

$$a_1 = 60 \quad S = \frac{60}{1 - (-\frac{1}{4})} = \boxed{48}$$

$r = -\frac{1}{4}$

h.  $\sum_{x=1}^5 (3x - 1) = \boxed{40}$

$$(3 \cdot 1 - 1) + (3 \cdot 2 - 1) + (3 \cdot 3 - 1) + (3 \cdot 4 - 1) + (3 \cdot 5 - 1)$$

i.  $\sum_{x=3}^9 (6x^2) = \boxed{1680}$

$$(6 \cdot 3^2) + (6 \cdot 4^2) + (6 \cdot 5^2) + (6 \cdot 6^2) + (6 \cdot 7^2) + (6 \cdot 8^2) + (6 \cdot 9^2)$$

7. A theater has 30 seats in the first row of the center section. Each row behind the first row gains two additional seats. How many seats are in the 5<sup>th</sup> row in the center section?

$$a_1 = 30 \quad a_5 = 30 + (4)2 = \boxed{38}$$

$d = 2$

$a_5 = ?$

8. Mr. Carlson suffers from allergies. When allergy season arrives, his doctor recommends that he take 300 mg of his medication the first day, and decrease the dosage by half each day for one week. What is the amount of medication Mr. Carlson will take on the 7<sup>th</sup> day?

$$a_1 = 300 \quad a_7 = 300 \left(\frac{1}{2}\right)^6 = \boxed{4.6875 \text{ mg}}$$

$r = \frac{1}{2}$

9. Lanie has decided to add strength training to her exercise program. Her trainer suggests that she add weight lifting for 5 minutes during her routine for the first week. Each week thereafter, she is to increase the weight lifting time by 2 minutes. How many minutes will she be devoting to weight lifting in week 10?

$$a_1 = 5 \quad a_{10} = 5 + (9)(2)$$

$d = 2$

$$\boxed{a_{10} = 23 \text{ min}}$$

$a_{10}$

10. There are 20 rows of seats on a concert hall: 25 seats are in the 1<sup>st</sup> row, 27 seats on the 2<sup>nd</sup> row, 29 seats on the 3<sup>rd</sup> row, and so on. If the price per ticket is \$2,300, how much will be the total sales for a one-night concert if all seats are taken?

$$a_1 = 25 \quad S_{20} = \frac{20}{2} (25 + a_{20}) \quad a_{20} = 25 + (19)(2) = 63$$

$d = 2$

$$S_{20} = 10 (25 + 63) = 880$$

$$\$2300 \cdot 880 = \boxed{\$2,024,000}$$

11. A tube well is bored 800 meters deep. The 1<sup>st</sup> meter costs \$250 and the cost per meter increases by \$50 for every subsequent meter. Find the total cost incurred for the entire job.

$$a_1 = 250$$

$$d = 50$$

$$S_{800} = \frac{800}{2}(250 + a_{800}) \quad a_{800} = 250 + (799)(50) = 40200$$

$$S_{800} = 400(250 + 40200)$$

$$\boxed{\$16,180,000}$$

12. You have won contest sponsored by a local radio station. If you are given the choice of the two payment plans. Plan A) \$1 on the first day, \$2 on the second day, \$3 on the third day, etc., for two weeks. Plan B) \$0.01 on the first day, \$0.02 on the second day, \$0.04 on the third day, etc. for two weeks. Which plan will pay you more? How much more?

13. Once a week Mrs. Baker makes sugar cookies. The first week she makes the recipe, she uses the full 2 cups of sugar called for. Each week after that, she reduces the amount of sugar by one third. How much sugar has she used for the cookies after five weeks.

$$a_1 = 2$$

$$r = \frac{1}{3}$$

$$S_5 = \frac{2(1 - \frac{1}{3}^5)}{1 - \frac{1}{3}} = \boxed{\frac{242}{81} \approx 3 \text{ cups}}$$

Plan A \$105

$$a_1 = 1$$

$$d = 1$$

$$S_{14} = \frac{14}{2}(a_1 + a_{14})$$

$$a_{14} = 1 + (13) \cdot 1 = 14$$

$$S_{14} = 7(1 + 14) = 105$$

Plan B

$$a_1 = 0.01$$

$$r = 2$$

$$S_{14} = \frac{0.01(1 - 2^{14})}{1 - 2} = \$163.83$$

Plan B. \$58.83 more