Sequences and Series Art Project

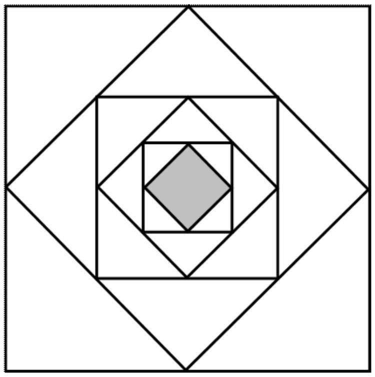
Choose at least one of the following projects to complete. Each project consists of a drawing portion and a math portion.

Drawing/Art – Follow the instructions for the project to create a visual representation of a sequence/series. Sketch out rough drafts to make sure you're on the right track. Once you are ready, take your time in creating a final product that incorporates color and creativity. While I do not expect all of you to be supreme artists, your product will be graded on accuracy and execution. Please create a product that you are proud of and will enjoy showing the class.

Mathematics – Answer the follow-up questions for your project. It may be helpful to read through the questions prior to starting your drawing and/or fill out the table as you go. You may work with others to check your answers, but you are all responsible for turning your math in with your project. If you run out of room on the sheet provided, you may continue on notebook paper.

Note: You may also choose to explore patterns/sequences/series/fractals that are not listed in this packet. If you choose a different project, you will need to show adequate amount of math (e.g. a table and recursive formula(s) for a property of your drawing).

Nested Squares



Draw a square. Assume this square has a perimeter of 40 ft. Connect the midpoints of the square with straight lines; the new figure will also be a square. Continue this process and you will create a series of nested squares. Determine the lengths of the sides of the squares and perimeters of your squares.

1) Complete the table below. The first square you drew corresponds to n = 1, the second square is n = 2, etc. .Hint: you will need to use the Pythagorean Theorem to find the length of the sides of the new squares.

n	1	2	3	4	5
Side Length of nth square (L_n)					
Side Length of nth square divided by 2 ($L_n/2$)					
Perimeter of nth square (P_n)					
nth partial sum of perimeters (S_n)					

2) Write a recursive formula for the perimeter of the nth square (P_n).

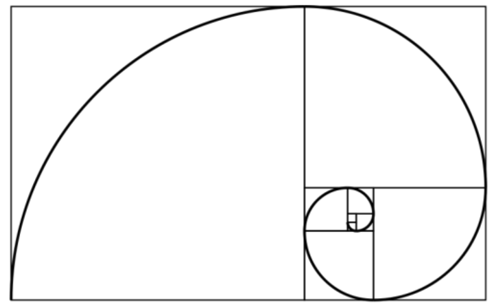
3) Write an explicit formula for the perimeter of the nth square (P_n).

4) Find the formula for the nth partial sum of the perimeters (S_n)

5) If the series for the perimeters of the square continues forever, what is the sum of the perimeters of all squares (S)?

Fibonacci Spiral

Draw two small squares. Each of these squares has a side length of 1. Adjacent to the two squares, draw a third square that has a side length of two (i.e. has the same length as the two small squares). Continue rotating your paper and drawing squares whose sides lengths follow the Fibonacci Sequence (1, 1, 2, 3, 5, 8, …). When you run out of room, start the original squares and draw a spiral outward. In each square you will be drawing a quarter circle; the circle's radius should match the side length of the square.



1) Write the recursive formula for the Fibonacci Sequence; you will need to specify the first two terms (1 and 1).

2) Complete the following table, where f_n is the nth term of the Fibonacci Sequence

n	1	2	3	4	5	6	7	8	9	10	11
f_n											
f_{n+1}											
f_{n+1}/f_n											

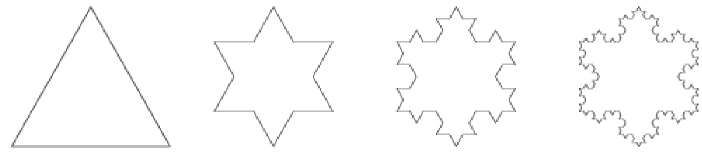
3) What value does f_{n+1} / f_n approach as n gets bigger? This value is the golden ratio.

4) Take the golden ratio and subtract 1. Find the reciprocal of the golden ratio. Notice anything?

5) Take the golden ratio and add 1. Square the golden ratio. Notice anything? Pretty cool, huh?

6) Draw a rectangle that has a short side that has a length of 1, and a long side with a length of the golden ratio. Do you find this rectangle visually appealing?

von Koch snowflake



Draw an equilateral triangle. Divide the sides of the triangle into thirds. Remove the middle third of each side. Add two additional line segments to each missing gap to form a smaller equilateral triangle. The first iteration of the snowflake should look like the Star of David. Continue this process until you cannot draw your triangles any smaller.

1) Complete the following table. Assume your first triangle had a perimeter of 9 inches.

n	1	2	3	4	5	6
Number of line segments (t_n)						
Length of each segment (L_n)						
Perimeter of snowflake (P_n)						

2) Write a recursive formula for the number of segments in the snowflake (t_n).

3) Write a recursive formula for the length of the segments (L_n).

4) Write a recursive formula for the perimeter of the snowflake (P_n).

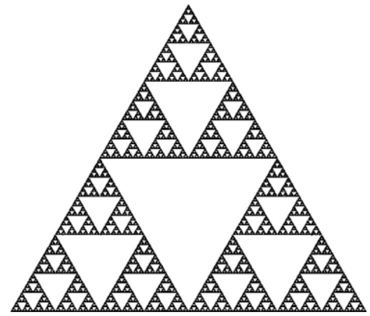
5) Write the explicit formulas for t_n, L_n, and P_n.

6) What is the perimeter of the infinite von Koch Snowflake?

7) Can you show why the area of the von Koch Snowflake is

$$\frac{81\sqrt{3}}{4}+\frac{27\sqrt{3}}{4}+3\sqrt{3}+\frac{4\sqrt{3}}{3}+\ldots+\frac{4^{n-3}\cdot\sqrt{3}}{3^{2n-7}}=\frac{162\sqrt{3}}{5}$$

Sierpenski's triangle



Draw an equilateral triangle. Connect the midpoints of your sides with straight lines, which should form a smaller inverted equilateral triangle in the middle of the original triangle. The triangle you just drew is "removed" from the original. Now in the remaining three equilateral potions, remove another set of triangles by connecting the midpoints. Continue until your sections are too small to remove accurately.

1) Complete the following table. Assume that your original triangle had an area of 100 cm^2 and that n =1 is the removal of the first triangle.

n	1	2	3	4	5	6
Number of triangles removed during iteration (t_n)						
Area of one of the removed triangles (A_n)						
Area removed during iteration (t_n x A_n)						
Total area remaining in Seirpenski's Triangle						
Total number of triangles removed (i.e. upside down triangles)						

2) Find a recursive formula for the area remaining in Seirpenski's Triangle.

3) What is the area of Seirpenski's Triangle after infinite iterations?

4) Find a recursive formula for the number of upside down triangles in Seirpenski's Triangle after n iterations.