NC MATH 3 HONORS UNIT 6 – EXPONENTIALS AND LOGARITHMS

6.1 PROPERTIES OF LOGARITHMS

Expand each logarithm.

1. $log_8 4ab^2$
2. $log_2(cd)^3$
3. $log_3 \frac{7}{v^3}$
4. $log \frac{w^5 x}{yz^9}$

Condense each expression into a single logarithm.

5. $log3 - log8$
6. $3log_4 x + log_4 y$
7. $log_5 2 + 6log_5 k - 3log_5 m$
8. $5log_3 x \cdot log_3 y$
9. $4(log_3 a + log_3 b)$
10. $2(log_9 2 + log_9 x) - 3(log_9 3 + log_9 y)$

Rewrite each equation in logarithmic form.

11. $3^5 = 243$
12. $81 = 243^{\frac{4}{5}}$

Rewrite each equation in exponential form.

13. $log_2 8 = 3$
14. $log_{243} 27 = \frac{3}{5}$

Evaluate each logarithm.

15. $log_5 125$
16. $log_{12} 12$
17. $\log 10^{-2}$
18. $log_7 7^8$
19. $log_{16} 1$
20. $\log 2$
21. $6^{log_6 7}$
22. $log_6 52$

6.2 Solving Exponential Equations

Solve each equation.

1. $7.5^{x} = 42.6$
2. $e^{4b} = 19$
3. $7 \cdot 5^{w} = 21$
4. $8^{h+3} = 12$
5. $6^{4p-1} = 18$
6. $9^{k-5} + 4 = 27$
7. $e^{x-1} - 5 = 5$
8. $3 \cdot 11^{2c+5} = 20$
9. $7^{m+4} = 5^{m}$
10. $6^{a} = 10^{a-2}$
11. $6^{2x+1} = 5^{4x-5}$
12. $2^{k+8} = 10^{k-4}$

6.3 Solving Logarithmic Equations

Solve each equation.

1. $log_3 x = 4$
2. $log_4(2x + 10) = 3$
3. $log_x 512 = 3$
4. $log_6(4x + 9) = log_6(2x + 19)$
5. $log(3x + 4) = 2$
6. $ln(2x + 4) = 3$
7. $log_3(3x - 6) = log_3(2x + 1)$
8. $log_7(3x + 7) = 4$

9. $lnx = 3$

10. $log_x 36 = 2$

11. $log_5(3x + 11) = 3$

12. $log_5 2 + log_5 x = 3$

13. $log_8 4x^4 - log_8 2x^2 = 1$

14. $log_2 x + log_2(x + 6) = 4$

15. $log_3(x + 10) - log_3 x = 4$

16. $log_7 x^2 = log_7(x + 20)$

17. $lnx + lnx^2 = 8$

18. $log_4(x + 4) + log_4(x + 64) = 4$

6.5 COMPOUND INTEREST

1. How long does it take $1425 to triple if it is invested at 4% interest compounded quarterly?

2. At what interest rate compounded continuously would you have to invest $350 to have $800 available in 5 years?

3. What amount must be invested at 5% interest compounded monthly to have $6000 available in 10 years?

4. At what interest rate compounded monthly would you have to invest $1300 to double your money in 7 years?

5. Emmet deposits $650 in a savings account with 8% interest compounded quarterly. Maggie deposits the same amount in another savings account with 8.2% interest compounded semiannually. If both Emmet and Maggie leave their money in the accounts for 2 years, which account will have the greater final balance?

6. If $800 is invested at 8% interest compounded continuously, how long will it take before the amount is $900?

7. A laptop purchased for $800 decreases in value by 20% each year. How long will it take before the laptop to be worth $350?

8. Hugo deposits $200 in a savings account with 0.3% interest compounded quarterly. Grace deposits the same amount in another savings account with 0.3% interest compounded semiannually. If both Hugo and Grace leave their money in the accounts for 3 years, which account will have the greater final balance?

6.6 More Applications of Exponents and Logarithms

1. The half-life of Cesium-137 is 30.2 years. If the initial mass of the sample is 15 kg, how much will remain after 151 years?

2. Myerstopia has a population of 6000. After 10 years, the population has increased exponentially to 7183 people. How many people will be living in Myerstopia after 23 years?

3. A loaf of bread that currently sells for $3.60 sold for $3.10 six years ago. At what rate has the cost of the loaf of bread increased each year?

4. A diamond ring currently worth $3000 increases in value by 8% each year. What is the value of the ring in 50 years?

5. Carbon-14 has a half-life of 5700 years. Find the age of a sample at which 22% of the radioactive nuclei originally present have decayed.

6. A population of 100 rabbits are living on an island. After one year, the rabbit population has increased exponentially to 500 rabbits. What will the population be after another 6 months?

7. Carbon-14 has a half-life of 5700 years. Consider a sample of fossilized wood that when alive would have contained 24g of C-14. It now contains 1.5g. How old is the sample?

8. The half-life of a radioactive element is 133 days, but your sample will not be useful to you after 65% of the radioactive nuclei originally present have disintegrated. About how many days can you use the sample?

6.7 Combinations and Compositions of Functions

If $f(x) = x^2 - 1$, $g(x) = 2x - 3$, and $h(x) = 1 - 4x$, find the following functions, as well as any values indicated.

1. $(f - g)(x) =$
2. $(f - g)(3) =$
3. $(f + h)(x) =$
4. $(f + h)(-2) =$
5. $(g \cdot h)(x) =$
6. $(g \cdot h)(4) =$
7. $\left(\frac{f}{g}\right)(x) =$
8. $\left(\frac{f}{g}\right)(-1) =$

Let $f(x) = 2x - 1$, $g(x) = 3x$, and $h(x) = x^2 + 1$. Compute the following:

9. $f\big(g(x)\big) =$
10. $(h \circ g)(x) =$
11. $h\big(f(9)\big) =$
12. $f\big(g(-3)\big) =$
13. $g\Big(f\big(h(-6)\big)\Big) =$

For #'s 14 & 15, $h(x) = (f \circ g)(x)$

14. Let $h(x) = \sqrt{x-5}$ and $f(x) = \sqrt{x}$, find $g(x)$.
15. Let $h(x) = (5x+1)^2 - (5x+1)$ and $f(x) = x^2 - x$, find $g(x)$.

6.8 Inverse Functions

Find the inverse.

1. $f = \{(1,-2),(-2,1),(0,7)\}$
2. $f = \{(-6,3),(8,2),(3,3)\}$
3. $f(x) = \frac{3x-1}{8}$
4. $f(x) = \frac{-3x}{5x-1}$
5. $f(x) = \sqrt[3]{x+5} + 2$
6. $f(x) = 5\sqrt{x-4}$
7. $f(x) = \frac{7x+9}{6}$
8. $f(x) = \frac{2x}{5x-5}$
9. $f(x) = (x+9)^3 - 5$
10. $f(x) = \frac{2-4x}{-4-x}$
11. $f(x) = 17x^2$
12. $f(x) = \frac{-3-x}{1-4x}$

Determine if f(x) and g(x) are inverses. Justify your answer.

13. $f(x) = x + 1$ and $g(x) = x - 1$
14. $f(x) = 2x + 1$ and $g(x) = \frac{1}{2}x - 1$
15. $f(x) = \frac{x+3}{8}$ and $g(x) = 8x + 3$
16. $f(x) = \frac{7x+5}{2}$ and $g(x) = \frac{2x-5}{7}$
17. $f(x) = (x+3)^2 - 2$ and $g(x) = \sqrt{x+2} - 3$