9. $\ln x = 3$

$e^3 = x$

$20.086 \approx x$

10. $\log_x 36 = 2$

$x^2 = 36$

$x = 6$

11. $\log_5(3x + 11) = 3$

$5^3 = 3x + 11$

$114 = 3x$ $\quad x = 38$

12. $\log_5 2 + \log_5 x = 3$

$\log_5(2x) = 3$ $\quad x = 62.5$

$5^3 = 2x$

$125 = 2x$

13. $\log_8 4x^4 - \log_8 2x^2 = 1$

$\log_8\left(\frac{4x^4}{2x^2}\right) = 1$

$\log_8(2x^2) = 1$

$8^1 = 2x^2$

$4 = x^2$

$x = \pm 2$

14. $\log_2 x + \log_2(x + 6) = 4$

$\log_2(x(x+6)) = 4$ $\quad x^2 + 6x - 16 = 0$

$2^4 = x(x+6)$ $\quad (x-2)(x+8) = 0$

$16 = x^2 + 6x$ $\quad x = 2, -8$

15. $\log_3(x + 10) - \log_3 x = 4$

$\log_3\left(\frac{x+10}{x}\right) = 4$ $\quad \frac{x+10}{x} = 81$ $\quad x = \frac{1}{8}$

$\frac{x+10}{x} = 3^4$ $\quad 81x = x + 10$

$80x = 10$

16. $\log_7 x^2 = \log_7(x + 20)$

$x^2 = x + 20$ $\quad x = 5, -4$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

17. $\ln x + \ln x^2 = 8$

$\ln(x \cdot x^2) = 8$ $\quad x^3 = e^8$

$\ln(x^3) = 8$ $\quad x = \sqrt[3]{e^8} \approx 14.392$

18. $\log_4(x + 4) + \log_4(x + 64) = 4$

$\log_4[(x+4)(x+64)] = 4$

$4^4 = (x+4)(x+64)$

$256 = x^2 + 68x + 256$

$0 = x^2 + 68x$

$0 = x(x+68)$

$x = 0, \ x = -68$

6.5 COMPOUND INTEREST

1. How long does it take $1425 to triple if it is invested at 4% interest compounded quarterly? $4275 = 1425\left(1 + \frac{0.04}{4}\right)^{4t}$ $\quad 3 = (1.01)^{4t}$ $\quad \ln 3 = \ln(1.01)^{4t}$ $\quad \ln 3 = 4t \ln(1.01)$ $\quad t = \frac{\ln 3}{4\ln(1.01)} = 27.602$ $\quad 27.602$ years

2. At what interest rate compounded continuously would you have to invest $350 to have $800 available in 5 years? $800 = 350e^{r5}$ $\quad \frac{16}{7} = e^{5r}$ $\quad \ln(16/7) = \ln e^{5r}$ $\quad \ln(16/7) = 5r$ $\quad r = \frac{\ln(16/7)}{5}$ $\quad r = 0.1653$ $\quad 16.5\%$

3. What amount must be invested at 5% interest compounded monthly to have $6000 available in 10 years? $6000 = P\left(1 + \frac{0.05}{12}\right)^{12(10)}$ $\quad 6000 = P(1.004166666667)^{120}$ $\quad P = 3642.9966$ $\quad \$3,642.97$

4. At what interest rate compounded monthly would you have to invest $1300 to double your money in 7 years? $2600 = 1300\left(1 + \frac{r}{12}\right)^{12(7)}$ $\quad 2 = \left(1 + \frac{r}{12}\right)^{84}$ $\quad \sqrt[84]{2} = \sqrt[84]{\left(1 + \frac{r}{12}\right)^{84}}$ $\quad \sqrt[84]{2} = 1 + \frac{r}{12}$ $\quad \sqrt[84]{2} - 1 = \frac{r}{12}$ $\quad 12\left(\sqrt[84]{2} - 1\right) = r$ $\quad r = 0.0994$ $\quad 9.94\%$

5. Emmet deposits $650 in a savings account with 8% interest compounded quarterly. Maggie deposits the same amount in another savings account with 8.2% interest compounded semiannually. If both Emmet and Maggie leave their money in the accounts for 2 years, which account will have the greater final balance?

E $A = 650\left(1+\frac{0.08}{4}\right)^{4\cdot 2}$ $A = 761.5785977$

M $A = 650\left(1+\frac{0.08}{2}\right)^{2\cdot 2}$ $A = 760.4080064$

Emmet $761.58

6. If $800 is invested at 8% interest compounded continuously, how long will it take before the amount is $900? $900 = 800e^{0.08(t)}$ $\frac{9}{8} = e^{0.08t}$ $\ln(9/8) = \ln e^{0.08t}$ $\ln(9/8) = 0.08t$ $t = \frac{\ln(9/8)}{0.08} = 1.47$ years

7. A laptop purchased for $800 decreases in value by 20% each year. How long will it take before the laptop to be worth $350? $350 = 800(1-\frac{0.2}{1})^{1t}$ $\frac{7}{16} = (0.8)^t$ $\ln(7/16) = \ln(0.8)^t$ $\ln(7/16) = t(\ln 0.8)$ $t = \frac{\ln(7/16)}{\ln(0.8)} = 3.70$ years

8. Hugo deposits $200 in a savings account with 0.3% interest compounded quarterly. Grace deposits the same amount in another savings account with 0.3% interest compounded semiannually. If both Hugo and Grace leave their money in the accounts for 3 years, which account will have the greater final balance?

H: $A = 200\left(1+\frac{0.003}{4}\right)^{4\cdot 3}$ $A = 201.8074436$ $201.81

G: $A = 200\left(1+\frac{0.003}{2}\right)^{2\cdot 3}$ $A = 201.8067635$ $201.81

The balances will be the same.

6.6 MORE APPLICATIONS OF EXPONENTS AND LOGARITHMS

1. The half-life of Cesium-137 is 30.2 years. If the initial mass of the sample is 15 kg, how much will remain after 151 years?

2. Myerstopia has a population of 6000. After 10 years, the population has increased exponentially to 7183 people. How many people will be living in Myerstopia after 23 years?

3. A loaf of bread that currently sells for $3.60 sold for $3.10 six years ago. At what rate has the cost of the loaf of bread increased each year?

4. A diamond ring currently worth $3000 increases in value by 8% each year. What is the value of the ring in 50 years?

5. Carbon-14 has a half-life of 5700 years. Find the age of a sample at which 22% of the radioactive nuclei originally present have decayed.

6. A population of 100 rabbits are living on an island. After one year, the rabbit population has increased exponentially to 500 rabbits. What will the population be after another 6 months?