5. Emmet deposits $650 in a savings account with 8% interest compounded quarterly. Maggie deposits the same amount in another savings account with 8.2% interest compounded semiannually. If both Emmet and Maggie leave their money in the accounts for 2 years, which account will have the greater final balance?

E: $A = 650\left(1+\frac{0.08}{4}\right)^{4\cdot 2}$ $A = 761.5785977$

M: $A = 650\left(1+\frac{0.082}{2}\right)^{2\cdot 2}$ $A = 760.4080064$

Emmet $761.58

6. If $800 is invested at 8% interest compounded continuously, how long will it take before the amount is $900? $900 = 800e^{0.08(t)}$ $\frac{9}{8} = e^{0.08t}$ $\ln(9/8) = \ln e^{0.08t}$ $\ln(9/8) = 0.08t$ $t = \frac{\ln(9/8)}{0.08} = 1.47$ years

7. A laptop purchased for $800 decreases in value by 20% each year. How long will it take before the laptop to be worth $350? $350 = 800(1-\frac{0.2}{1})^{1t}$ $\frac{7}{16} = (0.8)^t$ $\ln(7/16) = \ln(0.8)^t$ $\ln(7/16) = t(\ln 0.8)$ $t = \frac{\ln(7/16)}{\ln 0.8} = 3.70$ years

8. Hugo deposits $200 in a savings account with 0.3% interest compounded quarterly. Grace deposits the same amount in another savings account with 0.3% interest compounded semiannually. If both Hugo and Grace leave their money in the accounts for 3 years, which account will have the greater final balance?

H: $A = 200\left(1+\frac{0.003}{4}\right)^{4\cdot 3}$ $A = 201.8074436$ $\$201.81$

G: $A = 200\left(1+\frac{0.03}{2}\right)^{2\cdot 3}$ $A = 201.8067635$ $\$201.81$

The balances will be the same.

6.6 MORE APPLICATIONS OF EXPONENTS AND LOGARITHMS

1. The half-life of Cesium-137 is 30.2 years. If the initial mass of the sample is 15 kg, how much will remain after 151 years? $A = 15\left(\frac{1}{2}\right)^{151/30.2}$ $A = 0.46875$ 0.469 kg

2. Myerstopia has a population of 6000. After 10 years, the population has increased exponentially to 7183 people. How many people will be living in Myerstopia after 23 years? $7183 = 6000(b)^{10}$ $\frac{7183}{6000} = b^{10}$ $b = \sqrt[10]{\frac{7183}{6000}}$ $b = 1.018158665$ $y = 6000(1.018158665)^{23}$ $y = 9076.2587$ 9076 people

3. A loaf of bread that currently sells for $3.60 sold for $3.10 six years ago. At what rate has the cost of the loaf of bread increased each year? $3.60 = 3.10(b)^6$ $1.161290323 = b^6$ $b = \sqrt[6]{\frac{3.6}{3.1}} = 1.0252$ 2.52%

4. A diamond ring currently worth $3000 increases in value by 8% each year. What is the value of the ring in 50 years? $y = 3000(1.08)^{50}$ $y = 140704.8375$ $140,704.84

5. Carbon-14 has a half-life of 5700 years. Find the age of a sample at which 22% of the radioactive nuclei originally present have decayed. $88 = 100(\frac{1}{2})^{x/5700}$ $0.88 = (\frac{1}{2})^{x/5700}$ $\ln(0.88) = \ln(\frac{1}{2})^{x/5700}$ $\ln(0.88) = \frac{x}{5700}(\ln\frac{1}{2})$ $x = \frac{(\ln 0.88)\cdot 5700}{\ln(\frac{1}{2})}$ $x = 105.122$ 105.122 years

6. A population of 100 rabbits are living on an island. After one year, the rabbit population has increased exponentially to 500 rabbits. What will the population be after another 6 months? $500 = 100(b)^1$ $5 = b$ $y = 100(5)^{1.5}$ $y = 1118.033989$ 1118 rabbits

7. Carbon-14 has a half-life of 5700 years. Consider a sample of fossilized wood that when alive would have contained 24g of C-14. It now contains 1.5g. How old is the sample?

$1.5 = 24\left(\frac{1}{2}\right)^{x/5700}$ $0.0625 = \left(\frac{1}{2}\right)^{x/5700}$ $\ln .0625 = \ln\left(\frac{1}{2}\right)^{x/5700}$ $\ln .0625 = \frac{x}{5700}\ln\left(\frac{1}{2}\right)$ $x = \frac{(\ln 0.0625)\,5700}{\ln\left(\frac{1}{2}\right)} = 22800$ years

8. The half-life of a radioactive element is 133 days, but your sample will not be useful to you after 65% of the radioactive nuclei originally present have disintegrated. About how many days can you use the sample?

$65 = 100\left(\frac{1}{2}\right)^{x/133}$ $0.65 = \left(\frac{1}{2}\right)^{x/133}$ $\ln(0.65) = \ln(0.5)^{x/133}$ $\ln(0.65) = \frac{x}{133}\cdot\ln 0.5$ $x = \frac{\ln(0.65)\cdot 133}{\ln(0.5)}$ $x = 82.65795411$

82 days

6.7 COMBINATIONS AND COMPOSITIONS OF FUNCTIONS

If $f(x) = x^2 - 1$, $g(x) = 2x - 3$, and $h(x) = 1 - 4x$, find the following functions, as well as any values indicated.

1. $(f-g)(x) = (x^2-1) - (2x-3) = x^2 - 2x + 2$ ($x^2 - 1 - 2x + 3$)

2. $(f-g)(3) = (3)^2 - 2(3) + 2 = 5$

3. $(f+h)(x) = (x^2-1) + (1-4x) = x^2 - 4x$ ($x^2 - 1 + 1 - 4x$)

4. $(f+h)(-2) = (-2)^2 - 4(-2) = 12$

5. $(g\cdot h)(x) = (2x-3)(1-4x) = -8x^2 + 14x - 3$ ($2x - 8x^2 - 3 + 12x$)

6. $(g\cdot h)(4) = -8(4)^2 + 14(4) - 3 = -75$

7. $\left(\frac{f}{g}\right)(x) = \frac{x^2-1}{2x-3}$

8. $\left(\frac{f}{g}\right)(-1) = \frac{(-1)^2-1}{2(-1)-3} = \frac{0}{-5} = 0$

Let $f(x) = 2x - 1$, $g(x) = 3x$, and $h(x) = x^2 + 1$. Compute the following:

9. $f(g(x)) = 2(3x) - 1 = 6x - 1$

10. $(h \circ g)(x) = (3x)^2 + 1 = 9x^2 + 1$

11. $h(f(9)) = (2\cdot 9 - 1)^2 + 1 = 290$

12. $f(g(-3)) = 2(3\cdot -3) - 1$ $2(-9) + 1 = -17$

13. $g(f(h(-6))) = 219$
$h(-6) = (-6)^2 + 1 = 37$
$f(37) = 2(37) - 1 = 73$
$g(73) = 3\cdot 73 = 219$

For #'s 14 & 15, $h(x) = (f \circ g)(x)$

14. Let $h(x) = \sqrt{x-5}$ and $f(x) = \sqrt{x}$, find g(x). $g(x) = x - 5$

15. Let $h(x) = (5x+1)^2 - (5x+1)$ and $f(x) = x^2 - x$, find g(x). $g(x) = 5x + 1$