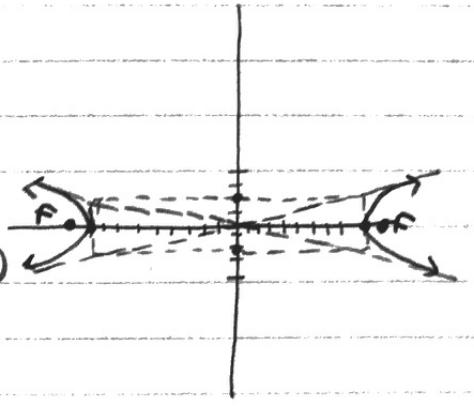
7.6 Hyperbolas

1. $\frac{x^2}{81} - \frac{y^2}{4} = 1$

opens left/right
center (0,0)
foci $(\sqrt{85}, 0)$ $(-\sqrt{85}, 0)$
vertices (-9,0) (9,0)
asymptotes
$y = \pm \frac{2}{9}x$

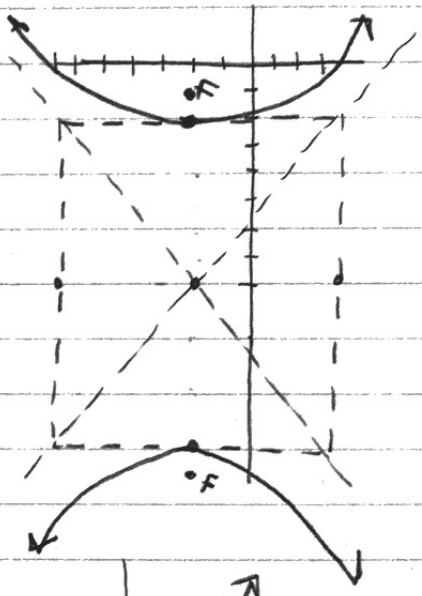
$a^2 = 81 \quad b^2 = 4$
$a = 9 \quad b = 2$
$c^2 = 81 + 4 = 85$
$c = \sqrt{85} \approx 9.22$



2. $\frac{(y+8)^2}{36} - \frac{(x+2)^2}{25} = 1$

opens up/down
center (-2,-8)
vertices (-2,-2) (-2,-14)
foci $(-2, -8+\sqrt{61})$ $(-2, -8-\sqrt{61})$
asymptotes
$y+8 = \pm \frac{6}{5}(x+2)$

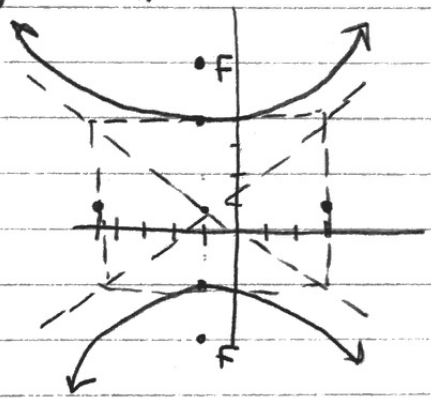
$a^2 = 36 \quad b^2 = 25$
$a = 6 \quad b = 5$
$c^2 = 36 + 25 = 61$
$c = \sqrt{61} \approx 7.810$



3. $\frac{(y-1)^2}{9} - \frac{(x+1)^2}{16} = 1$

opens up/down
center (-1,1)
foci
(-1,6) (-1,-4)
vertices
(-1,4) (-1,-3)

$a^2 = 9 \quad b^2 = 16$
$a = 3 \quad b = 4$
$c^2 = 9 + 16 = 25$
$c = 5$



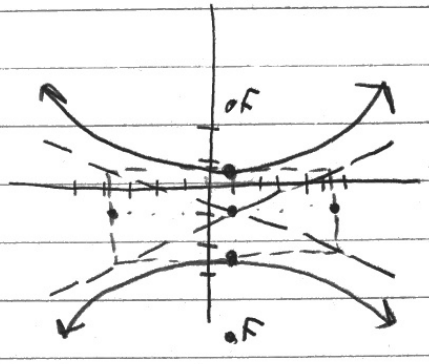
4. $9y^2 - x^2 + 2x + 54y + 62 = 0$

$9y^2 + 54y \quad -x^2 + 2x \quad = -62$

$9(y^2 + 6y + 9) - (x^2 - 2x + 1) = -62 + 81 - 1$

$\frac{9(y^2+3)^2}{18} - \frac{(x-1)^2}{18} = \frac{18}{18}$

$\frac{(y^2+3)^2}{2} - \frac{(x-1)^2}{18} = 1$



center $(1, -1)$

opens up/down

$a^2 = 2 \quad b^2 = 18$

$a = \sqrt{2} \approx 1.73 \quad b = \sqrt{18} = 3\sqrt{2} \approx 4.243$

$c^2 = 2 + 18 = 20$

$c = \sqrt{20} = 2\sqrt{5} \approx 4.472$

foci $(1, -1 + 2\sqrt{5})$

$(1, -1 - 2\sqrt{5})$

vertices $(1, -1 + \sqrt{2})$

$(1, -1 - \sqrt{2})$

asymptotes $y + 3 = \pm \frac{3\sqrt{2}}{\sqrt{2}}(x - 1)$

$\boxed{y + 3 = \pm 3(x - 1)}$

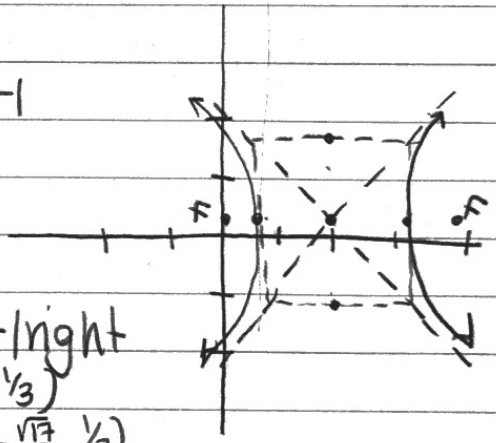
5. $9x^2 - 9y^2 - 36x - 6y + 18 = 0$

$9x^2 - 36x \quad -9y^2 - 6y \quad = -18$

$9(x^2 - 4x + 4) - 9(y^2 - \frac{2}{3}y + \frac{1}{9}) = -18 + 36 - 1$

$\frac{9(x-2)^2}{17} - \frac{9(y - \frac{1}{3})^2}{17} = \frac{17}{17}$

$\frac{(x-2)^2}{\frac{17}{9}} - \frac{(y - \frac{1}{3})^2}{\frac{17}{9}} = 1$



$a^2 = \frac{17}{9} \quad b^2 = 17/9$

$a = \frac{\sqrt{17}}{3} \quad b = \sqrt{17}/3 \approx 1.374$

$c^2 = \frac{17}{9} + \frac{17}{9} = \frac{34}{9}$

$c = \frac{\sqrt{34}}{3} \approx 1.944$

opens left/right

center $(2, \frac{1}{3})$

vertices $(2 + \frac{\sqrt{17}}{3}, \frac{1}{3})$

$(2 - \frac{\sqrt{17}}{3}, \frac{1}{3})$

foci $(2 + \frac{\sqrt{34}}{3}, \frac{1}{3})$

$(2 - \frac{\sqrt{34}}{3}, \frac{1}{3})$

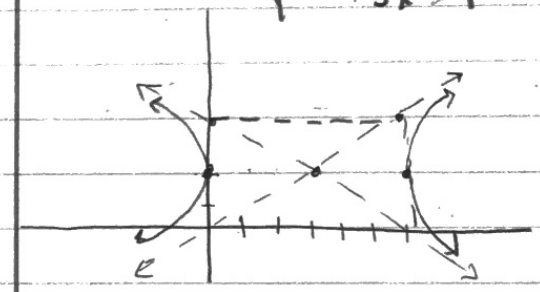
asymptotes

$y - \frac{1}{3} = \pm 1(x - 2)$

7. vertices (0,2) (6,2)

asymptotes $y = \frac{2}{3}x$

$y = -\frac{2}{3}x + 4$



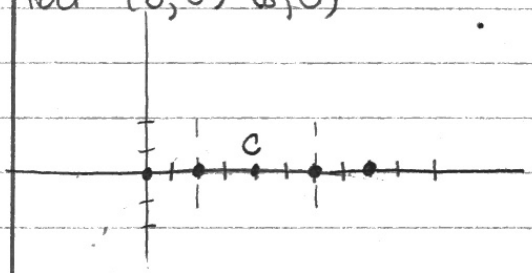
center (3,2)

$$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{4} = 1$$

8. whoops... repeat!

6. vertices (2,0) (6,0)

foci (0,0) (8,0)



center (4,0) opens L/R

$$\frac{(x-4)^2}{4} - \frac{y^2}{12} = 1$$

$a = 2 \quad c = 4$

$a^2 = 4 \quad c^2 = 16$

$c^2 = a^2 + b^2$

$16 = 4 + b^2$

$12 = b^2 \quad b = 2\sqrt{3}$