

Definition of a circle:

The set of all points equidistant from a fixed point (center)

When a circle has its center on the origin, then its equation is always of the form:

$$x^2 + y^2 = r^2$$

When a circle has its center somewhere other than the origin, then its equation is of the form:

$$(x-h)^2 + (y-k)^2 = r^2$$

Where (h, k) represent the coordinates of the center point.

For example, a circle which has center at (5, -3) and with a radius of 10, has the equation:

$$(x-5)^2 + (y+3)^2 = 100$$

Write an equation for each indicated circle:

1. center (1, 5), radius 12

$$(x-1)^2 + (y-5)^2 = 144$$

2. center (-4, 0), radius 5

$$(x+4)^2 + y^2 = 25$$

3. center (-7, -1), radius  $\sqrt{3}$

$$(x+7)^2 + (y+1)^2 = 3$$

4. center (-2, 6), radius  $\sqrt{7}$

$$(x+2)^2 + (y-6)^2 = 7$$

For each circle whose equation is given below, state the coordinates of the center point and state the radius:

5.  $(x-5)^2 + (y-7)^2 = 81$

C: (5, 7) r = 9

6.  $(x+4)^2 + (y+9)^2 = 64$

C: (-4, -9) r = 8

7.  $(x+3)^2 + y^2 = 16$

C: (-3, 0) r = 4

8.  $(x-6)^2 + (y+5)^2 = 12$

C: (6, -5) r =  $\sqrt{12} = 2\sqrt{3}$

Now suppose I have a circle with center point (5, -4). I don't know the radius but do know the circle goes through the point (2, -8). I want to find the equation of the circle:

Step 1: Use the Distance Formula to find the radius:

$$\text{Distance Formula } d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Step 2: Write the equation of the circle in the usual way:

$$(x-5)^2 + (y+4)^2 = 25$$

$$d = \sqrt{(5-2)^2 + (-4-(-8))^2}$$

$$d = \sqrt{9+16}$$

$$d = \sqrt{25} = 5$$

Find the equation of the circle that passes through the given point and has the given center:

9. center (5, -1) passes through the point (0, 11)

$$(x-5)^2 + (y+1)^2 = 169$$

$$r^2 = (5-0)^2 + (-1-11)^2$$

$$r^2 = 25 + 144 = 169$$

10. center (2, -3) passes through the point (6, -1)

$$(x-2)^2 + (y+3)^2 = 20$$

$$r^2 = (2-6)^2 + (-3-(-1))^2$$

$$r^2 = 16 + 4 = 20$$

When writing an equation in standard form, remember that a circle looks something like this:

$$(x-h)^2 + (y+k)^2 = 100$$

center: (4, -6) radius: 10

Consider the equation,  $x^2 + 8x + y^2 - 6y = 11$ . We can write the equation in standard form for a circle by completing the square, like so:

$$\text{Write: } x^2 + 8x + 16 + y^2 - 6y + 9 = 11 + 16 + 9$$

$$\frac{8}{2}(4)^2 = 16 \quad \frac{-6}{2}(-3)^2 = 9$$

Complete the square for the x-terms and for the y-terms. Balance your equations by adding the same numbers on the left and right side of the equal sign.

Now you have:  $(x+4)^2 + (y-3)^2 = 36$

Write this expression in standard form:  $(x+4)^2 + (y-3)^2 = 36$

Where is the center? (-4, 3) What is the radius? 6

Use completing the square to express each of the following as the equation of a circle in standard form. Then give the coordinates of the center point and the value of the radius.

11.  $x^2 - 12x + y^2 - 2y = -12$

$$(x-6)^2 + (y-1)^2 = 25$$

C: (6, 1) r = 5

12.  $x^2 + y^2 + 10y = 75$

$$x^2 + (y+5)^2 = 100$$

C: (0, -5) r = 10

13.  $x^2 + 4x + y^2 + 6y = 36$

$$(x+2)^2 + (y+3)^2 = 49$$

C: (-2, -3) r = 7

$$36 + 4 + 9 = 49$$