Pre-Calculus Parabola & Ellipse Applications Notes

The focus is a point on the axis of symmetry and a distance of "p" from the vertex on the inside the curve of the parabola.

The directrix is a line perpendicular to the axis of symmetry and a distance of "p" from the vertex on the outside of the curve of the parabola.

Ex. 1: Find the focus and directrix for each parabola.

a. $y = \frac{1}{2}x^2 - 2x + 3$

$\frac{1}{2}x^2 - 2x = y - 3$

$\frac{1}{2}(x^2 - 4x) = y - 3$

$\frac{1}{2}(x^2 - 4x + 4) = y - 3 + 2$

$\frac{1}{2}(x-2)^2 = y - 1$

$(x-2)^2 = 2(y-1)$

$4p = 2$

$p = 1/2$

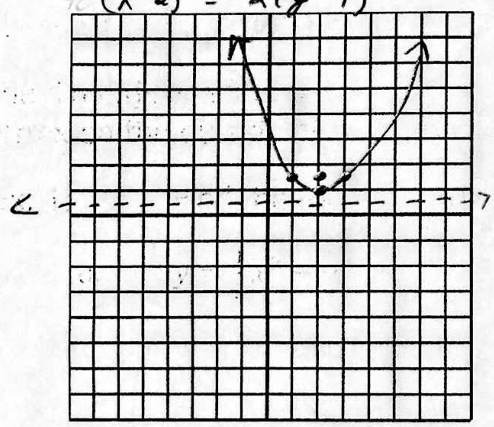
vertex $(2,1)$

opens up

focus $(2, 1.5)$

directrix: $y = 0.5$

LR: 2



b. $x = y^2 - 2y - 2$

$y^2 - 2y = x + 2$

$y^2 - 2y + 1 = x + 2 + 1$

$(y-1)^2 = x + 3$

$4p = 1$

$p = 1/4$

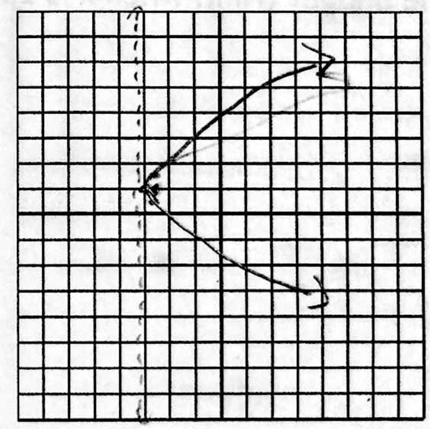
vertex $(-3,1)$

opens right

focus $(-2.75, 1)$

directrix: $x = -3.25$

LR = 1



Ex. 2: Write the equation of the parabola with the given characteristics.

a. focus: (0,4) and directrix is y = -4

opens up/down

$p = 4$ so $4p = 16$

vertex $(0,0)$

$(x-0)^2 = 16(y-0)^2$

$\boxed{x^2 = 16y^2}$

b. vertex: (-2,3) focus: (0,3)

opens right/left

$(y-3)^2 = 8(x+2)^2$

$p = 2$

so $4p = 8$

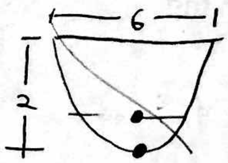
Ex. 3: Discuss the equation: $x^2 + 4x - 4y = 0$ Put the equation in standard form.

$x^2 + 4x = 4y$

$(x+2)^2 = 4(y+1)$

Applications of parabolas and ellipses

Ex. 4: A cable TV receiving dish is in the shape of a paraboloid of revolution. Find the location of the receiver, which is placed at the focus, if the dish is 6 feet across at its opening and 2 feet deep.



Ex. 5: The cables of a suspension bridge are in the shape of a parabola. The towers supporting the cable are 600 feet apart and 80 feet high. If the cables touch the road surface midway between the towers, what is the height of the cable at a point 150 feet from the center of the bridge? (Hint: when x is 150, what is y?)

Ex. 6: A bridge is built in the shape of a parabolic arch. The bridge has a span of 120 feet and a maximum height of 25 feet. Choose a suitable rectangular coordinate system and find the height of the arch of distances of 10, 30, and 50 feet from the center.

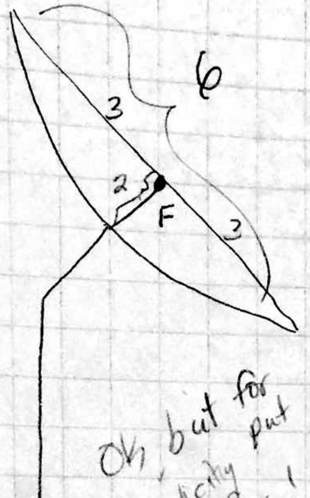
10 ft from center:________

30 ft from center:________

50 ft from center:________

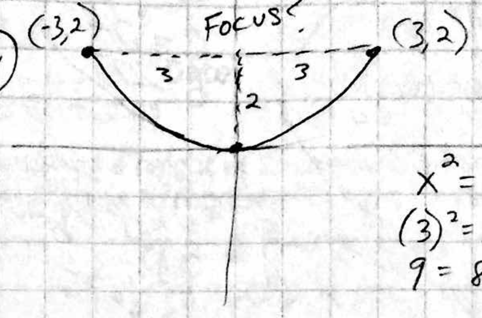
Ex. 7: A hall 100 feet in length is to be designed as a whispering gallery. If the foci are located 25 feet from the center, how high will the ceiling be at the center?

(4)



Ok, but for simplicity put @ origin!

(4)

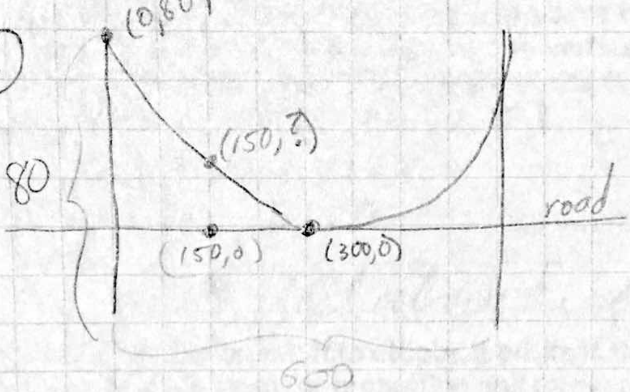


$4p = ?$

$x^2 = 4py$
$(3)^2 = 4p(2)$
$9 = 8p$
$p = 9/8$

so focus: $(0, 9/8)$

(5)



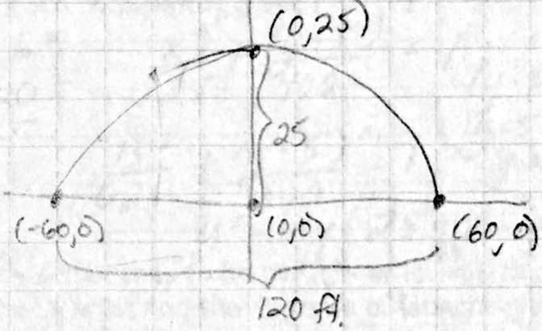
$x^2 = 4py$ vertex: $(300,0)$

$(x-300)^2 = 4py$ point: $(0,80)$
$(0-300)^2 = 4p(80)$
$90000 = 4p(80)$
$p = 281.25$ $4p = \frac{90000}{80} = 1125$

when $x = 150$:
$(150-300)^2 = 1125y$
$22500 = 1125y$

$y = 20$ ft.

(6)



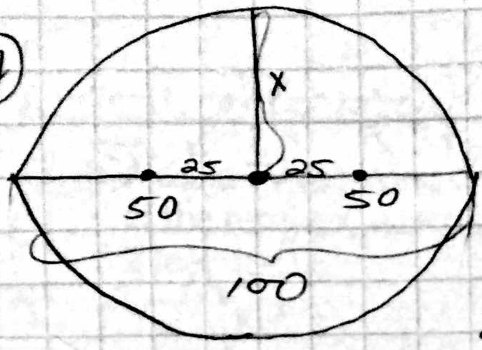
vertex: $(0,25)$

10 ft. from center?
30 ft. from center?
50 ft. from center?

$x^2 = 4py$, vertex $(0,25)$
$(x-0)^2 = 4p(y-25)$
$x^2 = 4p(y-25)$ point $(60,0)$
$(60)^2 = 4p(0-25)$
$4p = \frac{3600}{-25} = -144$

$x^2 = -144(y-25)$
$x=10$: $100 = -144(y-25)$ $y = 24.306$ ft
$x=30$: $900 = -144(y-25)$ $y = 18.75$ ft.
$x=50$: $2500 = -144(y-25)$ $y = 7.639$ ft.

⑦



$a = 50$

$c = 25$

$b = ?$

$c^2 = a^2 - b^2$

$25^2 = 50^2 - b^2$

$b^2 = 1875$

$b = \sqrt{1875} \approx \boxed{43.3 \text{ ft.}}$