

Unit 5 Bare Necessities - Rational Functions



Simplifying, Multiplying, and Dividing Rational Expressions

- When simplifying, factor completely then simplify
- When multiplying, multiply top to top, bottom to bottom, then simplify
- When dividing, remember to KEEP, CHANGE, FLIP!

$$1) \frac{3y(y+7)}{(y+7)(y^2-9)} = \frac{3y}{y^2-9}$$

$$2) \frac{p^2+2p-3}{p^2-2p-15} = \frac{(p+3)(p-1)}{(p-5)(p+3)} = \frac{p-1}{p-5}$$

$$3) \frac{x-5}{10x-2} \cdot \frac{25x^2-1}{x^2-10x+25}$$

$$\frac{x-5}{2(5x-1)} \cdot \frac{(5x-1)(5x+1)}{(x-5)(x+5)} = \frac{5x+1}{2(x+5)}$$

$$4) \frac{2d+6}{d^2+d-2} \div \frac{d+3}{d^2+3d+2}$$

$$\frac{2(d+3)}{(d+2)(d-1)} \cdot \frac{(d+2)(d+1)}{(d+3)} = \frac{2(d+1)}{d-1}$$

$$5) \frac{3x-9}{x-6} \div \frac{x^2-11x+24}{x^2-36}$$

$$\frac{3(x-3)}{x-6} \cdot \frac{(x-6)(x+6)}{(x-8)(x-3)} = \frac{3(x+6)}{x-8}$$

$$6) \frac{5x-9}{42x^3-48x^2} \cdot \frac{7x^2+27x-40}{45x^2-81x}$$

$$\frac{5x-9}{6x^2(7x-8)} \cdot \frac{(7x-8)(x+5)}{9x(5x-9)} = \frac{x+5}{54x^3}$$

Adding and Subtracting Rational Expressions - they must have the same denominator!

- Factor the bottoms and look for the LCD
- Multiply the top by what the numerator is missing from the LCD
- Rewrite the fractions with the same denominator as the LCD
- Add or subtract the numerators and be sure to combine like terms
- Rewrite the final numerator with the LCD as the denominator

$$7) \frac{5a^2}{6b} + \frac{9}{14a^2b^2} \quad \text{LCD} = 42a^2b^2$$

$$\frac{5a^2(7ab) + 9(3)}{42a^2b^2} = \frac{35a^4b + 27}{42a^2b^2}$$

$$8) \frac{5x}{x-3} + \frac{2}{x+4}$$

$$\frac{5x(x+4) + 2(x-3)}{(x-3)(x+4)} = \frac{5x^2 + 22x - 6}{(x-3)(x+4)}$$

$$9) \frac{x+10}{3x-15} - \frac{3x+15}{6x-30} \quad \text{LCD} = 6(x-5)$$

$$\frac{2(x+10) - (3x+15)}{6(x-5)} = \frac{-x+5}{6(x-5)} = \frac{-1(x-5)}{6(x-5)} = -\frac{1}{6}$$

$$10) \frac{5x}{x^2-x-6} - \frac{4}{x^2+4x+4} \quad \text{LCD} = (x-3)(x+2)(x+4)$$

$$\frac{5x(x+2) - 4(x-3)}{(x-3)(x+2)(x+4)} = \frac{5x^2 + 6x + 12}{(x-3)(x+2)(x+4)}$$

$$11) \frac{2r+6}{3r-6} + \frac{r+3}{3r-6}$$

$$\frac{3r+9}{3r-6} = \frac{3(r+3)}{3(r-2)} = \frac{r+3}{r-2}$$

$$12) \frac{5}{h+3} + \frac{5}{h^2-9} \quad \text{LCD} = (h+3)(h-3)$$

$$\frac{5(h-3) + 5}{(h+3)(h-3)} = \frac{5h-10}{(h+3)(h-3)}$$

Solving Rational Equations

- Factor all denominators and determine the LCM
- Multiply all parts of the equation by the LCM to eliminate all fractions
- Simplify each part of the equation and rewrite it
- Solve and check your answers!

$$13) \frac{7}{r+2} = \frac{6}{r-5}$$

$$7(r-5) = 6(r+2)$$

$$7r - 35 = 6r + 12$$

$$\boxed{r = 47}$$

$$15) p + \frac{p^2-5}{p^2-1} = \frac{p^2+p+2}{p+1}$$

$$(p+1)(p-1)$$

$$p(p+1)(p-1) + p^2-5 = (p-1)(p^2+p+2)$$

$$p(p^2-1) + p^2-5 = p^3+p^2+p-2$$

$$p^3+p^2-p-5 = p^3+p^2+p-2$$

$$p^3-2p-3=0$$

$$(p-3)(p+1)=0$$

$$\boxed{p=3} \quad p=-1 \text{ extraneous!}$$

$$17) \frac{2}{x+2} - \frac{1}{x} = \frac{-4}{x(x+2)}$$

$$2(x) - 1(x+2) = -4$$

$$2x - x - 2 = -4$$

$$x = -2$$

extraneous!

$$\boxed{\text{no solution}}$$

$$14) \frac{x}{x+2} - \frac{x+2}{x-2} = \frac{x+3}{x-2}$$

$$\frac{x}{x+2} = \frac{x+3}{x-2} + \frac{x+2}{x-2}$$

$$\frac{x}{x+2} = \frac{2x+5}{x-2}$$

$$x(x-2) = (2x+5)(x+2)$$

$$x^2-2x = 2x^2+9x+10$$

$$0 = x^2+11x+10$$

$$0 = (x+10)(x+1)$$

$$\boxed{x=-1 \quad x=-10}$$

$$16) \frac{3}{2x} - \frac{5}{3x} = 2 \quad \text{LCD} = 6x$$

$$3(3) - 5(2) = 2(6x)$$

$$-1 = 12x$$

$$\boxed{x = -\frac{1}{12}}$$

$$18) \frac{1}{2} = \frac{1}{x+3} + \frac{1}{x} \quad \text{LCD} = 2x(x+3)$$

$$x(x+3) = 2x + 2(x+3)$$

$$x^2+3x = 4x+6$$

$$x^2-x-6=0$$

$$(x-3)(x+2)=0$$

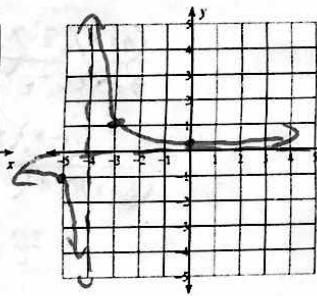
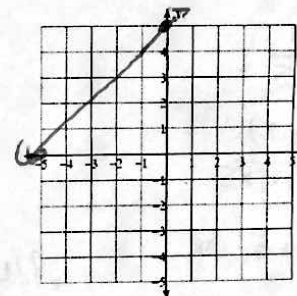
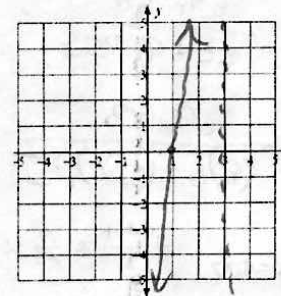
$$\boxed{x=3} \quad \boxed{x=-2}$$

Graphing Rational Functions

- Holes occur when the same factor "cancels out" of both the numerator and denominator
- Asymptotes are invisible lines that the graph approaches but never touches
- To find vertical asymptotes, set each factor in the denominator = 0 and solve
- To find horizontal asymptotes, compare the degrees of the numerator and denominator:
 - If $n < m$, then horizontal asymptote is $y = 0$.
 - If $n = m$, then horizontal asymptote is $y = \frac{a}{b}$.
 - If $n > m$, then there is no horizontal asymptote.
- Domain is the set of all possible x - values
- Range is the set of all possible y - values

$$19) f(x) = \frac{6(x-1)(x+9)(x-8)(x+7)}{(x+9)(x+7)(x-3)} \quad 20) f(x) = \frac{x^2+2x-15}{x-3} \quad 21) f(x) = \frac{x-1}{x^2+3x-4}$$

Holes:	$x = -9, x = -7$	$x = 3$	$x = 1$
Vertical Asymptotes:	$x = 3$	None	$x = -4$
Horizontal Asymptotes:	N/A (none)	None	$y = 0$
X-Intercepts:	$(8, 0) (1, 0)$	$(-5, 0)$	none
Y-Intercepts:	$(0, 48)$	$(0, 5)$	$(0, 1/4)$
Domain:	$(-\infty, -9) \cup (-9, -7) \cup (-7, 3) \cup (3, \infty)$	$(-\infty, 3) \cup (3, \infty)$	$(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$



Terrible grid for this!!