Unit 6 Bare Necessities - Exponential & Logarithmic Functions

Logarithmic Form **Exponential Form**

exponent

$\log_b x = y$ $b^y = x$

base

Write the following in exponential form:

1) $\log_3 \frac{1}{27} = -3$ 2) $\ln x = 2$ 3) $\log 3 = 1000$

$3^{-3} = \frac{1}{27}$ $e^2 = x$ $10^{1000} = 3$ what??

Write the following in logarithmic form:

4) $81 = 3^4$ 5) $2^5 = 32$ 6) $e^x = 3$

$\log_3 81 = 4$ $\log_2 32 = 5$ $\ln 3 = x$

Solving Logarithmic Equations

1. apply a property if needed to write as one log
2. convert to exponential form
3. solve for x

Properties

- $\log_b M + \log_b N = \log_b M \cdot N$
- $\log_b M - \log_b N = \log_b \frac{M}{N}$
- If $\log_b M = \log_b N$, then $M = N$

7) $\ln(3x-2) = 3$

$e^3 = 3x - 2$

$x = 7.3618$

8) $\log_5(x-2) + \log_5 2 = 1$

$5^1 = 2(x-2)$

$5 = 2x - 4$

$x = \frac{9}{2}$

9) $1 - 2\log(x+1) = -1$

$-2\log(x+1) = -2$

$\log(x+1) = 1$

$10^1 = x + 1$

$x = 9$

Solving Exponential Equations

1. Take the natural log of both sides
2. bring exponent down in front of the log
3. solve for x

10) $5^{1-2x} = 7$

$\ln 5^{1-2x} = \ln 7$

$(1-2x)\ln 5 = \ln 7$

$1 - 2x = \frac{\ln 7}{\ln 5}$

$x = -.1045$

11) $8^{4x+3} = 3^{2x-5}$

$\ln 8^{4x+3} = \ln 3^{2x-5}$

$\frac{(4x+3)\ln 8}{\ln 3} = \frac{(2x-5)\ln 3}{\ln 3}$

$1.8927(4x+3) = 2x - 5$

$5.5712x = -10.6784$

$x = -1.9167$

12) $e^{5x-2} = 40$

$\ln 40 = \ln e^{5x-2}$

$\frac{\ln 40 + 2}{5} = x$

$x = 1.1378$

Applications

Growth and Decay:

$y = a(b)^x$

a: initial amount

b: growth/decay factor

Compounded over time:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

A: final amount of money

P: initial amount of money

r: interest rate (as a decimal)

n: number of times compounded in a year

Compounded continuously:

$A = Pe^{rt}$

t: time (in years!)

13) a. Determine whether the function $y = 2(3.5)^x$ represents exponential growth or decay.

growth

b. Evaluate the function at f(x) = 4.

$y = 2(3.5)^4 = 300.125$

c. Find x when y = 75.

$75 = 2(3.5)^x$

$37.5 = 3.5^x$

$x = \frac{\ln 37.5}{\ln 3.5} = 2.8931$

14) Suppose a population of 15 zombies doubles in size every 6 months. Write a function that models this situation.

$y = 15(2)^{t/6}$ where t = months

a. How many zombies will there be after 2 years?

$y = 15(2)^{24/6} = 240$

b. How long will it take for the zombie population to reach 1,000?

$1000 = 15(2)^{t/6}$ 36.3531 months or 3.0294 yrs.

15) A car that costs $22,000 decreases in value 6% per year.

$y = 22{,}000(.94)^x$

a. How much will the car be worth in 4 years?

$y = 22{,}000(.94)^4 = \$17{,}176.48$

b. When will the car be worth $5,000?

$5000 = 22{,}000(.94)^x$ $x = 23.945$ yr.

16) Suppose you invest a total of $12,000 at an annual interest rate of 9%. Find the balance to the nearest dollar after 5 years if it is compounded:

a. Quarterly

$A = 12000\left(1 + \frac{.09}{4}\right)^{4(5)} = \$18{,}726.11$

b. Continuously

$A = 12000e^{.09(5)} = \$18{,}819.75$

Inverses

1. **Change f(x) to y**
2. **Switch x and y**
3. **Solve for y**
4. **Replace y with $f^{-1}(x)$**

17) $f(x) = -\frac{1}{4}x + 6$

$y = -\frac{1}{4}x + 6$

$x = -\frac{1}{4}y + 6$

$y = (x-6)(-4)$

$y = -4(x-6)$

$f^{-1}(x) = -4x + 24$

18) $f(x) = 2\sqrt{x} - 3$

$x = 2\sqrt{y} - 3$

$x + 3 = 2\sqrt{y}$

$\sqrt{y} = \frac{x+3}{2}$

$y = \left(\frac{x+3}{2}\right)^2 = f^{-1}(x)$

19) $f(x) = 2\ln(x+1) - 1$

$x = 2\ln(y+1) - 1$

$x + 1 = 2\ln(y+1)$

$\frac{x+1}{2} = \ln(y+1)$

$e^{\frac{x+1}{2}} - 1 = y = f^{-1}(x)$

26. Given $f(x) = 3x + 6$, $g(x) = 2x - 4$, *and* $h(x) = x^2 + 2$, *find*

a) $h(f(-1))$

$f(-1) = 3$

$h(3) = 11$

b) $(g \circ h)(4)$

$h(4) = 18$

$g(18) = 32$

c) $f(f(-3))$

$f(-3) = -3$

$f(-3) = -3$

d) $(h \circ g)(x - 1)$

$g(x-1) = 2(x-1) - 4$

$= 2x - 6$

$h(2x-6) = (2x-6)^2 + 2$

$= 4x^2 - 24x + 38$

Combinations of Functions:

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x) \cdot g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

20) Let $f(x) = 4x + 8$ and $g(x) = 2x - 12$, find:

a. $(f+g)(x)$

$6x - 4$

b. $(f-g)(x)$

$2x + 20$

c. $(f \cdot g\ (fg)(x)$

$8x^2 - 32x - 96$

~~d) $(\frac{f}{g})(x)$~~

d) $\frac{f}{g}(x)$

$\frac{4x+8}{2x-12} = \frac{4(x+2)}{2(x-6)} = \frac{2(x+2)}{x-6}$

24. Let $f(x) = x^2 - 9$ and $f(x) = x - 3$, find: a) (f + g)(x) b) (f - g)(x) c) (fg)(x) d) $(\frac{f}{g})(x)$

$x^2 + x - 12$

$x^2 - x - 6$

$x^3 - 3x^2 - 9x + 27$

$\frac{x^2-9}{x-3}$

$= \frac{(x+3)(x-3)}{(x-3)}$

$= \boxed{x+3}$

Function Operations - Combinations of Functions (Putting one function into another!):

- $(f \circ g)x \rightarrow f[g(x)]$ *read "f of g of x"*
- $(g \circ f)x \rightarrow g[f(x)]$ *read "g of f of x"*

25. Given $f(x) = x^2 + 1$ *and* $g(x) = x + 1$, *find*

a) $(f \circ g)(3)$

$g(3) = 4$

$f(4) = \boxed{17}$

b) $(g(f(-2))$

$f(-2) = 5$

$g(5) = 6$

c) $(g \circ f)(0)$

$f(0) = 1$

$g(1) = 2$

d) $f(g(x+5))$

$g(x+5) = x + 6$

$f(x+6) = (x+6)^2 + 1$

$= x^2 + 12x + 37$