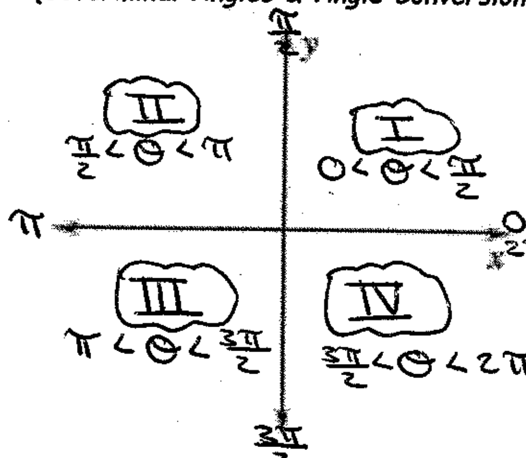
Precalculus Name <u>Key</u>

4.1 Notes: Radian and Degree Measure - *Day 2* *(Coterminal Angles & Angle Conversions)*

Summarize the range of angle measures (in radians) for each Quadrant in the diagram to right.

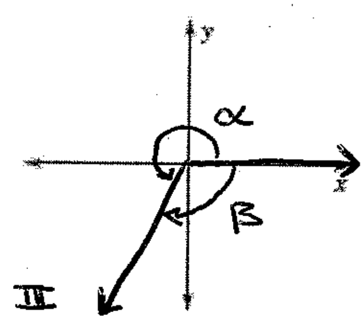


NOTE: If an angle is measured in degrees, you will see the degree symbol. Often, an angle measured in radians will have no units.

Angles are typically labeled in diagrams with <u>Greek letters</u> θ (theta), α (alpha), β (beta), etc.

We use these Greek letters as the variable to represent angles in diagrams, in formulas and in calculations.

In the diagram at the right, draw the terminal side of an angle in <u>Quadrant III.</u> Label the positive angle α and the negative angle β.



Notice how α and β have the <u>same terminal side</u> when drawn in standard position...

Remember...these angles are called <u>coterminal</u> angles.

EXAMPLE 2 - *Find coterminal angles* Answers may vary.

Find one positive and one negative angle that is coterminal with... * Infinite coterminal angles.

a. $\frac{5\pi}{6}$	b. $\frac{-3\pi}{4}$	c. $\frac{11\pi}{3}$
To find a positive angle with the same terminal side as $\frac{5\pi}{6}$, add one revolution of a circle or 2π. $\frac{5\pi}{6}+2\pi=\frac{5\pi}{6}+\frac{12\pi}{6}=\frac{17\pi}{6}$	$-\frac{3\pi}{4}+2\pi$ $-\frac{3\pi}{4}+\frac{8\pi}{4}=\frac{5\pi}{4}$ $\frac{5\pi}{4}+\frac{8\pi}{4}=\frac{13\pi}{4}$	$\frac{11\pi}{3}+2\pi$ $\frac{11\pi}{3}+\frac{6\pi}{3}=\frac{17\pi}{3}$
To find a negative angle with the same terminal side as $\frac{5\pi}{6}$, subtract one revolution of a circle or 2π. $\frac{5\pi}{6}-2\pi=\frac{5\pi}{6}-\frac{12\pi}{6}=\frac{-7\pi}{6}$	$-\frac{3\pi}{4}-\frac{8\pi}{4}=\frac{-11\pi}{4}$ $-\frac{11\pi}{4}-\frac{8\pi}{4}=\frac{-19\pi}{4}$	$\frac{11\pi}{3}-\frac{6\pi}{3}=\frac{5\pi}{3}$ * Subtract again! $\frac{5\pi}{3}-\frac{6\pi}{3}=-\frac{\pi}{3}$ $-\frac{\pi}{3}-\frac{6\pi}{3}=\frac{-7\pi}{3}$

Finding coterminal angles measured in degrees is much simpler - just add or subtract 360°.

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180°}$.

2. To convert radians to degrees, multiply radians by $\frac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship π rad = 180°. (See Figure 4.14.)

$$\frac{180°}{180} = \frac{\pi \text{ rad}}{180} \qquad \frac{\pi \text{ rad}}{\pi} = \frac{180°}{\pi}$$

$$1° = \frac{\pi \text{ rad}}{180°} \qquad 1 \text{ rad} = \frac{180°}{\pi}$$

$$1° \approx 0.0175 \text{ rad} \qquad 1 \text{ rad} \approx 57.296°$$

EXAMPLE 3 - *Convert between degree and radian units of measure*

Convert each angle measure into the "other" unit. Leave radian measures in terms of π.

a. $\theta = 75°$ $75°\left(\frac{\pi}{180°}\right)$ $\frac{75°\pi}{180°}$ $\frac{75\pi}{180}$ $\frac{5\pi}{12}$	b. $\theta = 320°$ $320°\left(\frac{\pi}{180°}\right)$ $\frac{320°\pi}{180°}$ $\frac{320\pi}{180}$ $\frac{16\pi}{9}$	c. $\theta = -45°$ $-45°\left(\frac{\pi}{180°}\right)$ $\frac{-45\pi}{180°}$ $\frac{-45\pi}{180}$ $-\frac{\pi}{4}$
d. $\theta = \frac{5\pi}{3}$ $\frac{5\cancel{\pi}}{\cancel{3}}\left(\frac{\overset{60°}{\cancel{180°}}}{\cancel{\pi}}\right)$ *cross Reduce $300°$	e. $\theta = \frac{-13\pi}{4}$ $\frac{-13\cancel{\pi}}{\cancel{4}}\left(\frac{\overset{45°}{\cancel{180°}}}{\cancel{\pi}}\right)$ $-585°$	f. $\theta = 3$ Radian measure (almost π) $3\left(\frac{180°}{\pi}\right)$ $\frac{540°}{\pi}$ $\approx 171.887°$