Precalculus   Name ___Key___

## 4.4 Notes: Trigonometric Functions of Any Angle-*Day 2*

Reference Angles
The values of the trigonometric functions of angles greater than 90° (or less than 0°) can be determined from their values at the corresponding acute angles called **reference angles**.

Definition of Reference Angle:

Let θ be an angle in ___Standard___ ___position___. Its **reference** angle is the ___acute___ ___angle___ θ' formed by the ___terminal___ ___side___ of θ and the ___x-axis___.

Note: reference angles are always __positive__.
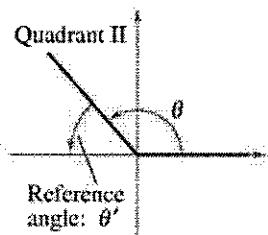
$\theta' = \pi - \theta$ (radians)
$\theta' = 180° - \theta$ (degrees)
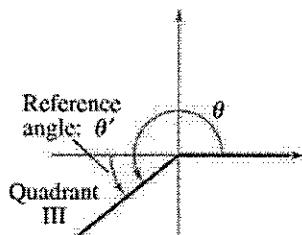
$\theta' = \theta - \pi$ (radians)
$\theta' = \theta - 180°$ (degrees)
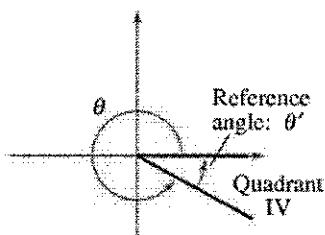
$\theta' = 2\pi - \theta$ (radians)
$\theta' = 360° - \theta$ (degrees)

Find the reference angle θ' and sketch θ and θ' in standard position.

1. $\theta = 300°$	2. $\theta = -135°$	3. $\theta = -870°$	4. $\theta = -292°$
$\theta' = 60°$	$\theta' = 45°$	$\theta' = 30°$	$\theta' = 68°$

$-870° + 360 + 360 = -150°$
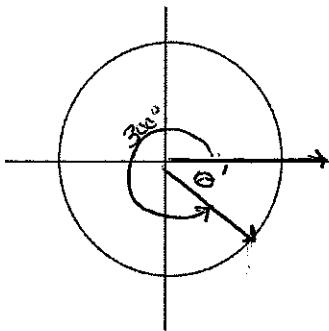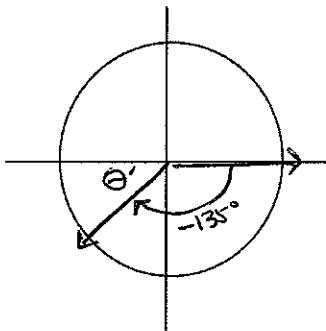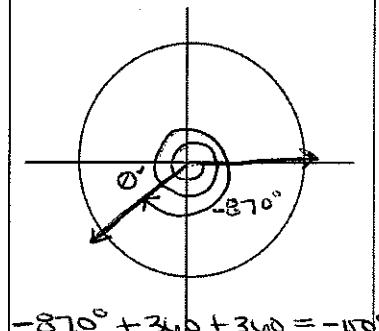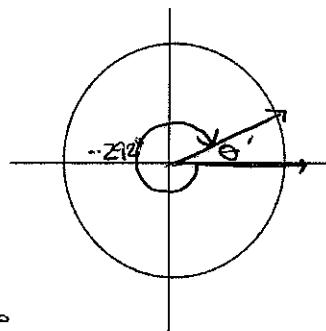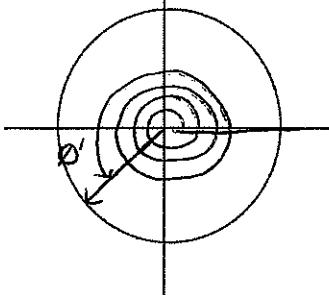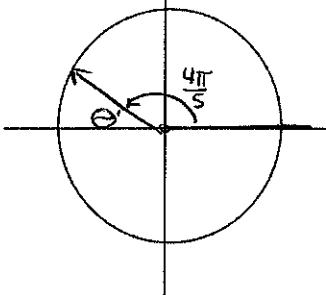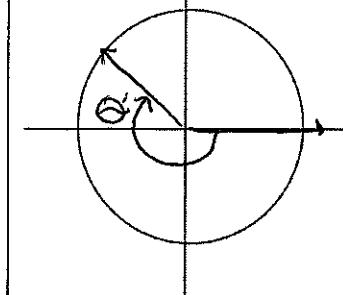
| 5. $\theta = \dfrac{51\pi}{7} = 7\frac{2}{7}\pi$	6. $\theta = \dfrac{4\pi}{5}$	7. $\theta = -\dfrac{11\pi}{9} = -1\frac{2}{9}\pi$	8. $\theta = 1.7$
$\theta' = \dfrac{2}{7}\pi$	$\theta' = \dfrac{\pi}{5}$	$\theta' = \dfrac{2}{9}\pi$	$\theta' = 1.442$
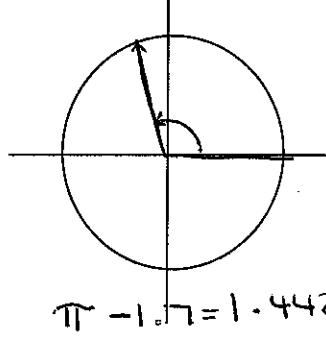
$\pi - 1.7 = 1.442$

To find the value of a trigonometric function of any angle θ:
- ➤ Determine the function value for the associated reference angle θ'.
- ➤ Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

Evaluate the sine, cosine, and tangent of each angle without using a calculator.

9. $\theta = -330°$ QUAD I $\theta' = 30°$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\sin\theta = \frac{1}{2}$ $\cos\theta = \frac{\sqrt{3}}{2}$ $\tan\theta = \frac{\sqrt{3}}{3}$	10. $\theta = 225°$ QUAD III $\theta' = 45°$ $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\sin\theta = -\frac{\sqrt{2}}{2}$ $\cos\theta = -\frac{\sqrt{2}}{2}$ $\tan\theta = 1$
11. $\theta = \frac{-17\pi}{6}$ $-2\frac{5}{6}\pi$ $\theta' = \frac{\pi}{6}$ $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ QUAD III $\sin\theta = \frac{-1}{2}$ $\cos\theta = -\frac{\sqrt{3}}{2}$ $\tan\theta = \sqrt{3}/3$	12. $\theta = \frac{19\pi}{4}$ $4\frac{3}{4}\pi$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ Quad II $\sin\theta = \frac{\sqrt{2}}{2}$ $\cos\theta = -\frac{\sqrt{2}}{2}$ $\tan\theta = -1$
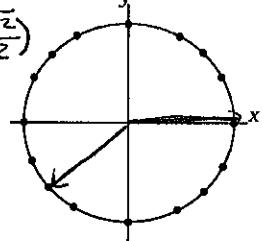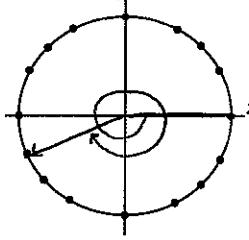
Find the indicated trigonometric value in the specified quadrant.

13. If $\sin\theta = -\frac{3}{5}$ and the angle is in quadrant IV, then find $\cos\theta$. $\cos\theta = \frac{4}{5}$	14. If $\csc\theta = -2$ and the angle is in quadrant IV, then find $\cot\theta$. $x^2 + (-1)^2 = 2^2$ $\tan\theta = \frac{-1}{\sqrt{3}}$ $x^2 = 3$ $x = \sqrt{3}$ $\cot\theta = -\sqrt{3}$	15. If $\sec\theta = -\frac{9}{4}$ and the angle is in quadrant III, then find $\tan\theta$. $(-4)^2 + y^2 = 9^2$ $16 + y^2 = 81$ $y^2 = 65$ $y = -\sqrt{65}$ $\tan\theta = \frac{\sqrt{65}}{4}$
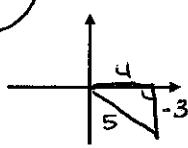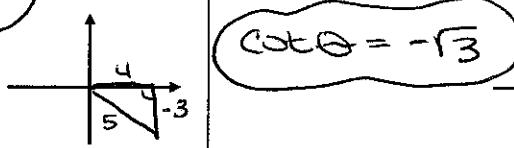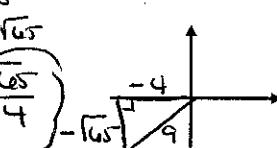
Find TWO solutions of the equation. Give you answers in degrees $(0° \le \theta < 360°)$ and radians $(0 \le \theta < 2\pi)$. Do not use your calculator.
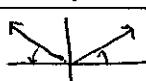
16. $\theta' = 30°$ $\sin\theta = \frac{1}{2}$ $\theta = 30°$ or $\frac{\pi}{6}$ $\theta = 150°$ or $\frac{5\pi}{6}$	17. $\theta' = 30°$ $\sin\theta = -\frac{1}{2}$ $\theta = 210°$ or $\frac{7\pi}{6}$ $\theta = 330°$ or $\frac{11\pi}{6}$	18. $\csc\theta = \frac{2\sqrt{3}}{3}$ $\sin\theta = \frac{\sqrt{3}}{2}$ $\theta' = 60°$ $\theta = 60°$ or $\frac{\pi}{3}$ $\theta = 120°$ or $\frac{2\pi}{3}$
19. $\cot\theta = -1$ $\tan\theta = -1$ $\theta' = 45°$ $\theta = 135°$ or $\frac{3\pi}{4}$ $\theta = 315°$ or $\frac{7\pi}{4}$	20. $\sec\theta = -\frac{2\sqrt{3}}{3}$ $\cos\theta = \frac{-\sqrt{3}}{2}$ $\theta' = 30°$ $\theta = 150°$ or $\frac{5\pi}{6}$ $\theta = 210°$ or $\frac{7\pi}{6}$	21. $\cos\theta = -\frac{1}{2}$ $\theta' = 60°$ $\theta = 120°$ or $\frac{2\pi}{3}$ $\theta = 240°$ or $\frac{4\pi}{3}$