Precalculus Name Key

5.2 Notes: Verifying Trigonometric Identities

Verifying Trigonometric Identities Algebraically: DO NOT CROSS THE EQUAL SIGN! Since you are trying to PROVE that both sides of the given equation are equal, you cannot use any properties of equality because that would mean that you already assume that they are equal.

Strategies:

- ***Work with the more complicated side of the equation and make it match the simpler side.***
- ***Simplify both sides until they match.***
- ***Look to factor.***
- ***Rewrite everything in terms of sine and cosine.***
- ***Look for a substitution.***
- ***Perform indicated operation (add, subtract, etc…)***
- ***Rationalizing***

Examples: Verify each identity algebraically.

1. $\dfrac{\sin^2 x - 1}{\sin^2 x} = -\cot^2 x$

*factor out −1

$\dfrac{-1(1-\sin^2 x)}{\sin^2 x} =$

$-\dfrac{\cos^2 x}{\sin^2 x} =$

$-\cot^2 x = -\cot x$

2. $\cot x \cdot \cos x = \dfrac{1}{\tan x \sec x}$

$= \dfrac{1}{\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}}$

$= \dfrac{\cos x}{\sin x} \cdot \dfrac{\cos x}{1}$

$\cot x \cdot \cos x = \cot x \cdot \cos x$

3. $\dfrac{\cot^2 \theta}{1+\csc\theta} = \dfrac{1-\sin\theta}{\sin\theta}$

$\dfrac{(1-\csc\theta)}{(1-\csc\theta)} \cdot \dfrac{\cot^2\theta}{(1+\csc\theta)} =$

$\dfrac{(1-\csc\theta)\cot^2\theta}{1-\csc^2\theta} =$

$\dfrac{(1-\csc\theta)\cancel{\cot^2\theta}}{-\cancel{\cot^2\theta}} =$

$-1 + \dfrac{1}{\sin\theta} =$

$\dfrac{-\sin\theta + 1}{\sin\theta} \rightarrow \dfrac{1-\sin\theta}{\sin\theta} = \dfrac{1-\sin\theta}{\sin\theta}$

4. $1-\sin^4 x = \cos^2 x\left(2-\cos^2 x\right)$

$= (1-\sin^2 x)(2-(1-\sin^2 x))$

$= (1-\sin^2 x)(1+\sin^2 x)$

$1-\sin^4 x = 1-\sin^4 x$

#3 $\dfrac{\csc^2\theta - 1}{1+\csc\theta} = \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta}$

*Diff of squares

$\dfrac{\cancel{(\csc\theta+1)}(\csc\theta-1)}{\cancel{(1+\csc\theta)}} = \csc\theta - 1$

$\csc\theta - 1 = \csc\theta - 1$

* common denominator

5. $\dfrac{\sec^2\theta-1}{\sec^2\theta}=\sin^2\theta$

xsplit fraction

$$\frac{\tan^2\theta}{\sec^2\theta} =$$

$$\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} =$$

$$\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cdot\frac{\cancel{\cos^2\theta}}{1} =$$

$$\frac{\sec^2\theta}{\sec^2\theta}-\frac{1}{\sec^2\theta}$$

$$1-\cos^2\theta$$

$$\sin^2\theta$$

$$\sin^2\theta=\sin^2\theta$$

6. $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=2\sec^2 x$

$$\frac{1+\sin x+1-\sin x}{(1-\sin x)(1+\sin x)} =$$

$$\frac{2}{1-\sin^2 x} =$$

$$\frac{2}{\cos^2 x} =$$

$$2\sec^2 x = 2\sec^2 x$$

7. $\sec^2 y-\tan^2 y=\tan y\cot y$

$$(1+\tan^2 y)-\tan^2 y = \cancel{\tan y}\cdot\frac{1}{\cancel{\tan y}}$$

$$1=1$$

$$=\tan y\cot y$$

$$=\tan y\cdot\frac{1}{\tan y}$$

$$=1$$

8. $\sec^4 A-\sec^2 A=\dfrac{1}{\cot^4 A}+\dfrac{1}{\cot^2 A}$

$$=\tan^4 A+\tan^2 A$$

$$=\tan^2 A(\tan^2 A+1)$$

$$=(\sec^2 A-1)(\sec^2 A-1+1)$$

$$=(\sec^2 A-1)(\sec^2 A)$$

$$\sec^4 A-\sec^2 A=\sec^4 A-\sec^2 A$$