Precalculus Name Key

5.4 Notes: Sum and Difference Identities-*Day 1*

Trigonometric Identities and Formulas to date:

The Reciprocal Identities:	**The Quotient Identities:**
$\sin x = \frac{1}{\csc x}$ $\csc x = \frac{1}{\sin x}$ $\cos x = \frac{1}{\sec x}$ $\sec x = \frac{1}{\cos x}$ $\tan x = \frac{1}{\cot x}$ $\cot x = \frac{1}{\tan x}$	$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$
The Pythagorean Identities:	**Sum & Difference Formulas:**
$\sin^2 x + \cos^2 x = 1$ $1+\tan^2 x = \sec^2 x$ $1+\cot^2 x = \csc^2 x$	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Investigation: Use a graphing utility to make a conclusion about the following questions.

1. Is it true that $\cos(x+2) = \cos x + \cos 2$?
2. Is it true that $\sin(x-4) = \sin x - \sin 4$?
3. Is it true that $\cos 75° = \cos 45° + \cos 30°$?

Evaluating a Trig Function:

Find the exact value of each function using the **sum and difference formulas**. In order to do this, you must select angles for which you know the EXACT values. $\left(ie: \sin 30^o = \frac{1}{2} \right)$

1. $\cos 75°$	2. $\tan 105°$
$\cos(45°+30°)$ $\cos45°\cos30° - \sin45°\sin30°$ $\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$ $\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$ $\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$	$\tan(60°+45°)$ $\frac{\tan60° + \tan45°}{1-\tan60°\tan45°}$ $\frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}$ $\frac{\sqrt{3}+1}{1-\sqrt{3}}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}}$ $\frac{\sqrt{3}+3+1+\sqrt{3}}{1-3} = \frac{4+2\sqrt{3}}{-2}$ $= \boxed{-2-\sqrt{3}}$

Sum & Difference Formulas: $\frac{\pi}{6} = \frac{2\pi}{12}$ $\frac{\pi}{3} = \frac{4\pi}{12}$ $\frac{\pi}{4} = \frac{3\pi}{12}$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Continue to find the exact value of the following functions.

3. $\sin\left(\frac{\pi}{12}\right)$ $\frac{4\pi}{12} - \frac{3\pi}{12}$

$\frac{\pi}{3} - \frac{\pi}{4}$

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

4. $\tan\left(\frac{11\pi}{12}\right)$ $\frac{3\pi}{12} + \frac{8\pi}{12}$ or $\frac{2\pi}{12} + \frac{9\pi}{12}$

$$\tan\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)$$

$$\frac{\tan\frac{\pi}{4} + \tan\frac{2\pi}{3}}{1 - \tan\frac{\pi}{4}\tan\frac{2\pi}{3}}$$

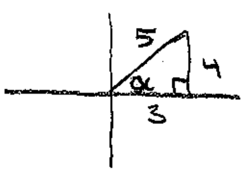
$$\frac{1 + -\sqrt{3}}{1 - (1)(-\sqrt{3})}$$

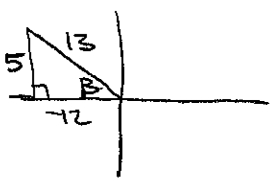
$$\frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{1 - 2\sqrt{3} + 3}{1 - 3}$$

$$= \frac{4 - 2\sqrt{3}}{-2} = \boxed{-2 + \sqrt{3}}$$

5. If $\sin\alpha = \frac{4}{5}$, where $0 < \alpha < \frac{\pi}{2}$ and $\cos\beta = -\frac{12}{13}$, where $\frac{\pi}{2} < \beta < \pi$

a) Find the exact value of $\sin(\alpha - \beta)$. (You can use triangles.)





$$\sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$$

$$\frac{-48}{65} - \frac{15}{65}$$

$$\boxed{-\frac{63}{65}}$$

b) Find the exact value of $\tan(\alpha - \beta)$. (You can use triangles.)

$$\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \frac{\frac{4}{3} - \left(-\frac{5}{12}\right)}{1 + \left(\frac{4}{3}\right)\left(-\frac{5}{12}\right)}$$

$$= \frac{\frac{4}{3} + \frac{5}{12}}{\frac{36}{36} - \frac{20}{36}} = \frac{\frac{16}{12} + \frac{5}{12}}{\frac{16}{36}} = \frac{21}{12} \cdot \frac{36}{16} = \boxed{\frac{63}{16}}$$