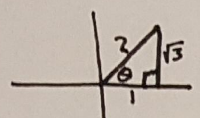
Honors Pre-Calculus

Double-Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$	$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

1. We can use Double-Angle formulas to evaluate various trigonometric ratios. Just be sure to DRAW YOUR TRIANGLES!

   a. Given $\cos\theta = \frac{1}{2}$ in Quadrant I, find $\cos(2\theta)$ and $\sin(2\theta)$.



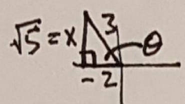
$1^2 + x^2 = 2^2$
$x^2 = 4 - 1$
$x^2 = 3$
$x = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$
$\cos\theta = \frac{1}{2}$

$\cos(2\theta) = 2\cos^2\theta - 1$
$= 2\left(\frac{1}{2}\right)^2 - 1$
$= 2\left(\frac{1}{4}\right) - 1$
$= \frac{1}{2} - 1$
$= \frac{1}{2} - \frac{2}{2} = \boxed{\frac{-1}{2}}$

$\sin(2\theta) = 2\sin\theta\cos\theta$
$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$
$= 2\left(\frac{\sqrt{3}}{4}\right)$
$= \boxed{\frac{\sqrt{3}}{2}}$

   b. Given $\boxed{\cos\theta = \frac{-2}{3}}$ in Quadrant II, find $\cos(2\theta)$ and $\tan(2\theta)$.



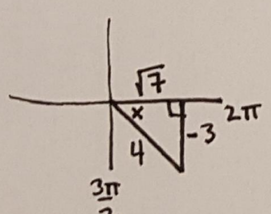
$(-2)^2 + (x)^2 = 3^2$ $\cos\theta$
$4 + x^2 = 9$
$x^2 = 5$
$x = \sqrt{5}$

$\tan\theta = \frac{\sqrt{5}}{-2}$

$\cos(2\theta) = 2\cos^2\theta - 1$
$= 2\left(\frac{-2}{3}\right)^2 - 1$
$= 2\left(\frac{4}{9}\right) - 1$
$= \frac{8}{9} - 1$
$= \boxed{-\frac{1}{9}}$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$
$= \dfrac{2\left(\frac{\sqrt{5}}{-2}\right)}{1-\left(\frac{\sqrt{5}}{-2}\right)^2}$
$= \dfrac{\frac{2\sqrt{5}}{-2}}{1-\frac{5}{4}} = \dfrac{\frac{2\sqrt{5}}{-2}}{-\frac{1}{4}}$ $\quad \frac{4}{4} = 1$
$= -\frac{2\sqrt{5}}{2} \cdot \frac{-4}{1} = \frac{8\sqrt{5}}{2} = \boxed{4\sqrt{5}}$

   c. Given $\sin x = -\frac{3}{4}$ and $\frac{3\pi}{2} < x < 2\pi$, find $\sin(2\theta)$ and $\tan(2\theta)$.



$(-3)^2 + y^2 = 4^2$
$9 + y^2 = 16$
$y^2 = 7$
$y = \sqrt{7}$

$\sin x = \frac{-3}{4}$
$\cos x = \frac{\sqrt{7}}{4}$
$\tan x = \frac{-3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{-3\sqrt{7}}{7}$

$\sin(2\theta) = 2\sin\theta\cos\theta$
$= 2\left(\frac{-3}{4}\right)\left(\frac{\sqrt{7}}{4}\right)$
$= \frac{-6\sqrt{7}}{16}$
$= \boxed{\frac{-3\sqrt{7}}{8}}$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$
$= \dfrac{2\left(\frac{-3\sqrt{7}}{7}\right)}{1-\left(\frac{-3\sqrt{7}}{7}\right)^2}$
$= \dfrac{\left(\frac{-6\sqrt{7}}{7}\right)}{1-\left(\frac{63}{49}\right)}$
$= \dfrac{\left(\frac{-6\sqrt{7}}{7}\right)}{\frac{49}{49}-\frac{63}{49}} = \dfrac{\left(\frac{-6\sqrt{7}}{7}\right)}{-\frac{2}{7}}$
$= \frac{-6\sqrt{7}}{7} \cdot \frac{-7}{2} = \frac{42\sqrt{7}}{14} =$
$= \boxed{3\sqrt{7}}$

3. More verifying identities. Now you can use sum/difference formulas and double-angle formulas.

   a. Verify: $1-\cos(2x)\sec^2 x = \tan^2 x$

$1-(2\cos^2 x-1)\sec^2 x =$

$1-(2\cos^2 x\sec^2 x - \sec^2 x)=$

$1 - 2\cos^2 x\sec^2 x + \sec^2 x =$

$1 - \frac{2\cancel{\cos^2 x}}{1}\cdot\frac{1}{\cancel{\cos^2 x}} + \sec^2 x =$

$1 - 2 + \sec^2 x =$

$-1 + \sec^2 x = \tan^2 x$ ✓

   b. Verify: $\cos(x-\pi) = -\cos x$

$\cos x\cos\pi + \sin x\sin\pi = -\cos x$

$\cos x(-1) + \sin x(0) = -\cos x$

$-\cos x = -\cos x$ ✓

   c. Verify: $\frac{2\tan x}{1+\tan^2 x} = \sin(2x)$

$\frac{2\tan x}{\sec^2 x} = 2\sin x\cos x$

$\frac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$

$\frac{2\sin x}{\cancel{\cos x}}\cdot\frac{\cos^{\cancel{2}} x}{1} =$

$2\sin x\cos x =$ ✓

4. We can also solve equations involving trigonometric ratios.

   a. $\sin x = \frac{\sqrt{2}}{2}$

   b. $\cos x + 1 = 0$