

Honors Pre-Calculus

Double-Angle Formulas

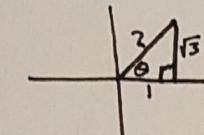
$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta\end{aligned}$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

1. We can use Double-Angle formulas to evaluate various trigonometric ratios. Just be sure to DRAW YOUR TRIANGLES!

- a. Given $\cos \theta = \frac{1}{2}$ in Quadrant I, find $\cos(2\theta)$ and $\sin(2\theta)$.



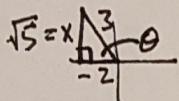
$$\begin{aligned}1^2 + x^2 &= 2^2 \\ x^2 &= 4 - 1 \\ x^2 &= 3 \\ x &= \sqrt{3}\end{aligned}$$

$$\begin{aligned}\sin \theta &= \frac{\sqrt{3}}{2} \\ \cos \theta &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\cos(2\theta) &= 2 \cos^2 \theta - 1 \\ &= 2 \left(\frac{1}{2}\right)^2 - 1 \\ &= 2 \left(\frac{1}{4}\right) - 1 \\ &= \frac{1}{2} - 1 \\ &= \frac{1}{2} - \frac{2}{2} = \boxed{-\frac{1}{2}}\end{aligned}$$

$$\begin{aligned}\sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{2}\right) \\ &= 2 \left(\frac{\sqrt{3}}{4}\right) \\ &= \boxed{\frac{\sqrt{3}}{2}}\end{aligned}$$

- b. Given $\cos \theta = -\frac{2}{3}$ in Quadrant II, find $\cos(2\theta)$ and $\tan(2\theta)$.



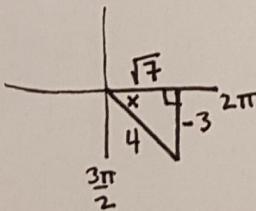
$$\begin{aligned}(-2)^2 + (x)^2 &= 3^2 \\ x^2 &= 9 - 4 \\ x^2 &= 5 \\ x &= \sqrt{5}\end{aligned}$$

$$\tan \theta = \frac{\sqrt{5}}{-2}$$

$$\begin{aligned}\cos(2\theta) &= 2 \cos^2 \theta - 1 \\ &= 2 \left(-\frac{2}{3}\right)^2 - 1 \\ &= 2 \left(\frac{4}{9}\right) - 1 \\ &= \frac{8}{9} - 1 \\ &= \boxed{-\frac{1}{9}}\end{aligned}$$

$$\begin{aligned}\tan(2\theta) &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2 \left(\frac{\sqrt{5}}{-2}\right)}{1 - \left(\frac{\sqrt{5}}{-2}\right)^2} \\ &= \frac{\frac{2\sqrt{5}}{-2}}{1 - \frac{5}{4}} = \frac{\frac{2\sqrt{5}}{-2}}{-\frac{1}{4}} \\ &= \frac{2\sqrt{5}}{2} \cdot \frac{-4}{1} = \frac{8\sqrt{5}}{2} = \boxed{4\sqrt{5}}\end{aligned}$$

- c. Given $\sin x = -\frac{3}{4}$ and $\frac{3\pi}{2} < x < 2\pi$, find $\sin(2x)$ and $\tan(2x)$.



$$\begin{aligned}(-3)^2 + y^2 &= 5^2 \\ y^2 &= 25 - 9 \\ y^2 &= 16 \\ y &= 4\end{aligned}$$

$$\sin x = -\frac{3}{5}$$

$$\begin{aligned}\cos x &= \frac{4}{5} \\ \tan x &= \frac{-3}{4} \cdot \frac{4}{5} = -\frac{3}{5}\end{aligned}$$

$$\begin{aligned}\sin(2x) &= 2 \sin x \cos x \\ &= 2 \left(-\frac{3}{5}\right) \left(\frac{4}{5}\right) \\ &= -\frac{24}{25} \\ &= \boxed{-\frac{24}{25}}\end{aligned}$$

$$\begin{aligned}\tan(2x) &= \frac{2 \tan x}{1 - \tan^2 x} \\ &= \frac{2 \left(-\frac{3}{5}\right)}{1 - \left(-\frac{3}{5}\right)^2} \\ &= \frac{\left(-\frac{6}{5}\right)}{1 - \left(\frac{9}{25}\right)} \\ &= \frac{\left(-\frac{6}{5}\right)}{\frac{16}{25}} = \frac{\left(-\frac{6}{5}\right)}{\frac{16}{25}} = \frac{(-6)(5)}{16} = -\frac{30}{16} = -\frac{15}{8} \\ &= -\frac{6\sqrt{7}}{7} \cdot -\frac{7}{2} = \frac{42\sqrt{7}}{14} = \boxed{3\sqrt{7}}\end{aligned}$$

3. More verifying identities. Now you can use sum/difference formulas and double-angle formulas.

a. Verify: $1 - \underbrace{\cos(2x)}_{\text{sec}^2 x} = \tan^2 x$

$$\begin{aligned} 1 - (2\cos^2 x - 1) \sec^2 x &= \\ 1 - (2\cos^2 x \sec^2 x - \sec^2 x) &= \\ 1 - 2\cos^2 x \sec^2 x + \sec^2 x &= \\ 1 - \cancel{2\cos^2 x} \cdot \frac{1}{\cos^2 x} + \sec^2 x &= \\ 1 - 2 + \sec^2 x &= \\ -1 + \sec^2 x &= \tan^2 x \quad \checkmark \end{aligned}$$

b. Verify: $\cos(x - \pi) = -\cos x$

$$\begin{aligned} \cos x \cos \pi + \sin x \sin \pi &= -\cos x \\ \cos x (-1) + \sin x (0) &= -\cos x \\ -\cos x &= -\cos x \quad \checkmark \end{aligned}$$

c. Verify: $\frac{2 \tan x}{1 + \tan^2 x} = \sin(2x)$

$$\begin{aligned} \frac{2 \sin x}{\cos x} &\stackrel{\frac{2 \tan x}{\sec^2 x}}{=} 2 \sin x \cos x \\ \frac{2 \sin x}{\cos x} \cdot \frac{\cos^2 x}{1} &= \\ 2 \sin x \cos x &= \quad \checkmark \end{aligned}$$

4. We can also solve equations involving trigonometric ratios.

a. $\sin x = \frac{\sqrt{2}}{2}$

b. $\cos x + 1 = 0$