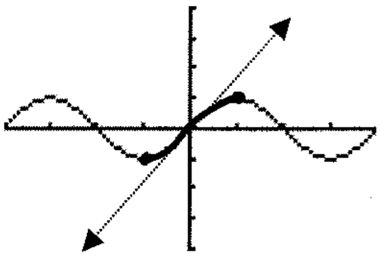
Precalculus Name Key

4.7 Notes: Inverse Trigonometric Functions-*Day 1*

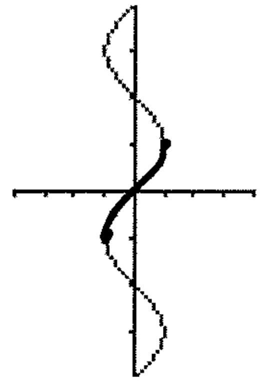
Graphing inverse functions:

Definition of a **function**: (must pass the vertical line test) every element of the domain (x) is paired with exactly one element of the range (y).

Recall that the inverse of a function can be graphed by reflecting points over the $y = x$ line.



Reflected over $y = x$



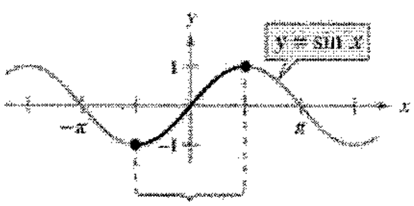
$y = \sin x$

$y = \sin^{-1} x$

Is the reflected graph a function? Why or why not?
No, fails the vertical line test

In order to create a function, we must **limit the domain**. If the domain is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then the inverse will also be a function. **Highlight** the restricted domain on the graph above. The graph of $y = \sin^{-1} x$ will be a function as long as it has this restricted domain. By restricting the domain of each trigonometric function, we can create an inverse trigonometric function.

$y = \sin x$



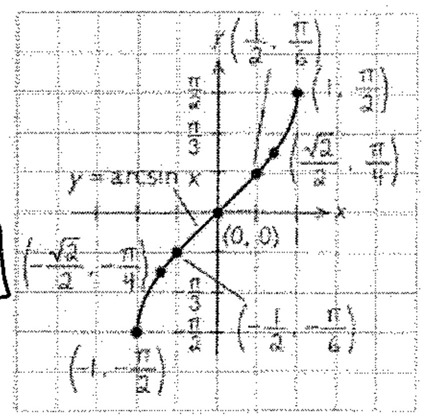
Sin x has an inverse function on this interval.

Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Range: $[-1, 1]$

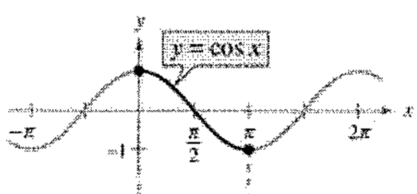
$y = \sin^{-1} x$

Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$y = \cos x$



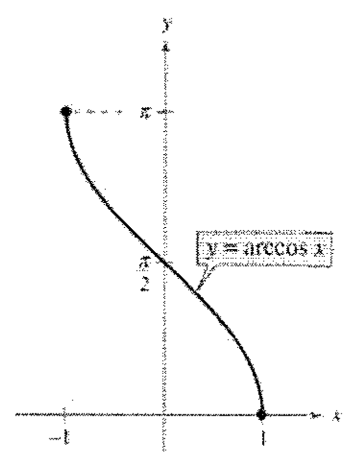
Cos x has an inverse function on this interval.

Domain: $[0, \pi]$ Range: $[-1, 1]$

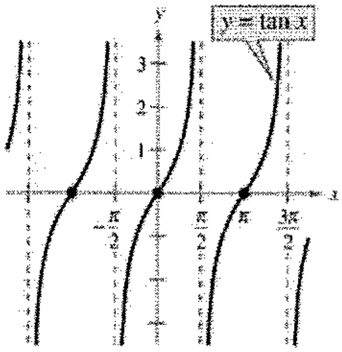
$y = \cos^{-1} x$

Domain: $[-1, 1]$

Range: $[0, \pi]$



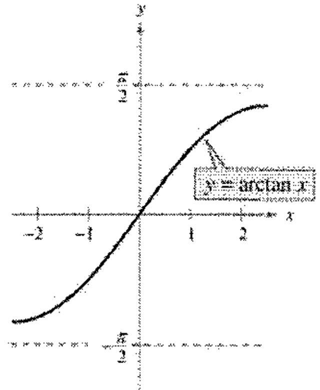
$y = \tan x$



Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range: $(-\infty, \infty)$

$y = \tan^{-1} x$



Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

The inverse trigonometric functions are denoted two ways:

$y = \arcsin x$		$y = \sin^{-1} x$
$y = \arccos x$	or	$y = \cos^{-1} x$
$y = \arctan x$		$y = \tan^{-1} x$

$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

When evaluating an inverse trigonometric function such as the arcsine, remember that the "arcsine of x is the angle whose sine is x". You are determining the **ANGLE**. Also, you are only to give answers on the restricted **ranges** for each inverse trigonometric function. List these ranges below. *MEMORIZE these ranges!*

$y = \sin^{-1} x$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (quad I + IV) $\quad$ $y = \cos^{-1} x$ $[0, \pi]$ (quad I, II) $\quad$ $y = \tan^{-1} x$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (quad I, IV)

Examples: Find the exact value in radian measure without using a calculator.

1. $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ $\sin\theta = -\frac{1}{2}$	2. $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ $\sin ? = \frac{\sqrt{2}}{2}$	3. $\sin^{-1}(2)$ undefined $\sin ? = 2$ *not in the domain
4. $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ $\cos ? = \frac{\sqrt{3}}{2}$	5. $\cos^{-1}(-5)$ undefined $\cos ? = -5$ *not in the domain	6. $\cos^{-1}(-1) = \pi$ $\cos ? = -1$
7. $\tan^{-1}(0) = 0$ $\tan ? = 0$	8. $\tan^{-1}(-1) = -\frac{\pi}{4}$ $\tan ? = -1$	9. $\arctan(\sqrt{3}) = \frac{\pi}{3}$ $\tan ? = \sqrt{3}$
10. $\arcsin(-1) = -\frac{\pi}{2}$ $\sin ? = -1$	11. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ $\cos ? = \frac{1}{2}$	12. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ $\tan ? = \frac{\sqrt{3}}{3}$

Examples: Use a calculator to approximate the value in radian measure (if possible). Round values to the nearest ten-thousandth.

13. $\tan^{-1}(-8.45)$ ≈ -1.4530	14. $\arcsin(0.2447)$ ≈ 0.2472	15. $\arccos(2)$ undefined (not in the domain)