Precalculus Name Key

5.3 Notes: Solving Trigonometric Equations

Identities are true for all values of *x* and we use them to simply other trig identities.

Equations are true only for specific values of *x*. When we solve an equation, we are finding those specific values of *x* that satisfy the equation.

- The primary goal in solving trigonometric equations is to isolate the trig function involved in the equation. Then, consider the values for "x" that make that equation true.
- If there is no restriction on domain, then state all the possible values by determining the first solution and then the interval by which all other solutions will occur. Use "n" to represent any integer.

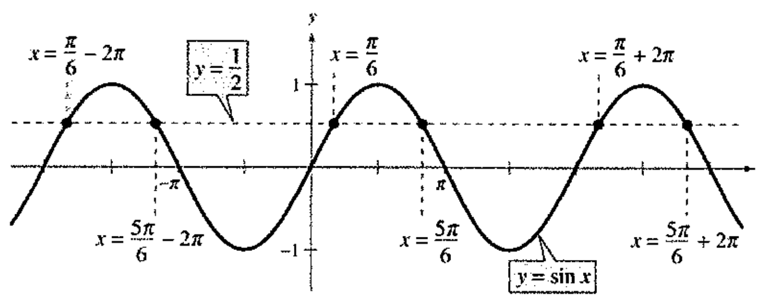
Solve over the interval $\left[0,2\pi\right)$.

1. $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$



2. $\sin^2 x = 2\sin x$

$\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$

$\sin x = 0$ $\quad$ $\sin x - 2 = 0$

$\boxed{x = 0, \pi}$ $\quad$ $\sin x = 2$

$\varnothing$

3. $\csc^2 x - 2 = 0$

$\csc^2 x = 2$

$\sqrt{\csc^2 x} = \sqrt{2}$

$\csc x = \pm\sqrt{2}$ $\quad$ * all quadrants

$\sin x = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

4. $\left(\sqrt{3}\tan x - 1\right)\left(\tan x - \sqrt{3}\right) = 0$

$\sqrt{3}\tan x - 1 = 0$ $\quad$ $\tan x - \sqrt{3} = 0$

$\sqrt{3}\tan x = 1$ $\quad$ $\tan x = \sqrt{3}$

$\tan x = \frac{1}{\sqrt{3}}$ $\quad$ $\boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$

$\tan x = \frac{\sqrt{3}}{3}$

$\boxed{x = \frac{\pi}{6}, \frac{7\pi}{6}}$

5. $2\sin^2 x + 3\cos x - 3 = 0$

$2(1 - \cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$-2\cos^2 x + 3\cos x - 1 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x - 1 = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = 1$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ $\quad$ $\boxed{x = 0}$

Solve for all real numbers.

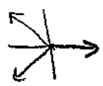
6. $\cos x+\sqrt{2}=-\cos x$

$2\cos x+\sqrt{2}=0$

$2\cos x=-\sqrt{2}$

$\cos x=\frac{-\sqrt{2}}{2}$

$\boxed{x=\frac{3\pi}{4}+2\pi n \quad x=\frac{5\pi}{4}+2\pi n}$



7. $\cot x\cos^2 x=2\cot x$

$\cot x\cos^2 x-2\cot x=0$

$\cot x\,(\cos^2 x-2)=0$

$\cot x=0 \qquad \cos^2 x-2=0$

$\cos^2 x=2$

$\cos x=\pm\sqrt{2}$

$\varnothing$

$\boxed{x=\frac{\pi}{2}+n\pi}$

8. $4\sin^2 x-3=0$

$4\sin^2 x=3$

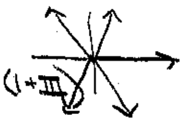
$\sin^2 x=\frac{3}{4}$

$\sin x=\pm\sqrt{\frac{3}{4}}$

$\sin x=\pm\frac{\sqrt{3}}{2}$

*all quadrants

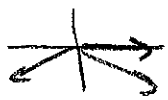
$\boxed{x=\frac{\pi}{3}+n\pi \quad x=\frac{2\pi}{3}+n\pi}$ (I+III) (II, IV)



For functions of the form $\sin kx$: solve for (kx), then divide by k.

Solve for all real numbers.

9. $\sin(2x)=-\frac{1}{2}$



let $u=2x$

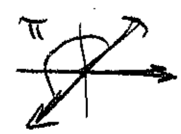
$\sin u=-\frac{1}{2}$

$u=\frac{7\pi}{6}+2\pi n, \quad u=\frac{11\pi}{6}+2\pi n$

$2x=\frac{7\pi}{6}+2\pi n \quad 2x=\frac{11\pi}{6}+2\pi n$

$\boxed{x=\frac{7\pi}{12}+\pi n, \quad x=\frac{11\pi}{12}+\pi n}$

10. $\tan\left(\frac{x}{2}\right)-1=0$



let $u=\frac{x}{2}$

$\tan u-1=0$

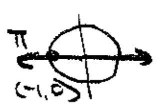
$\tan u=1$

$u=\frac{\pi}{4}+\pi n$

$\frac{x}{2}=\frac{\pi}{4}+\pi n$

$\boxed{x=\frac{\pi}{2}+2\pi n}$

11. $\cos 2x=-1$



let $u=2x$

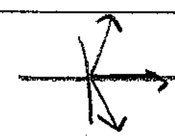
$\cos u=-1$

$u=\pi+2\pi n$

$2x=\pi+2\pi n$

$\boxed{x=\frac{\pi}{2}+\pi n}$

12. $\sec 4x=2$



let $u=4x$

$\sec u=2$

$\cos u=\frac{1}{2}$

$u=\frac{\pi}{3}+2\pi n, \quad u=\frac{5\pi}{3}+2\pi n$

$4x=\frac{\pi}{3}+2\pi n, \quad 4x=\frac{5\pi}{3}+2\pi n$

$\boxed{x=\frac{\pi}{12}+\frac{n\pi}{2}, \quad x=\frac{5\pi}{12}+\frac{n\pi}{2}}$