Polynomial Functions Review Key

1. Complete the table below

	Function	Degree	End Behavior	Domain and Range
A	$f(x) = 3x^5 - x^{10}$	5	$x \to -\infty\ f(x) \to -\infty$ $x \to \infty\ f(x) \to \infty$	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$
B	$g(x) = -x^2 + 5x + 3$	2	$x \to -\infty\ f(x) \to -\infty$ $x \to \infty\ f(x) \to -\infty$	D: $(-\infty, \infty)$ R: $(-\infty, 9.25]$
C	$h(x) = 3(x + 2)(x - 4)$	2	$x \to -\infty\ f(x) \to \infty$ $x \to \infty\ f(x) \to \infty$	D: $(-\infty, \infty)$ R: $[-27, \infty)$
D	$j(x) = -2x^3 - x^2 + 5x - 1$	3	$x \to -\infty\ f(x) \to \infty$ $x \to \infty\ f(x) \to -\infty$	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

2. Evaluate the polynomial $f(x) = 3x^5 - x^3 + 6x^2 - x + 1$ for $x = -2$. Explain what your answer represents. $f(-2) = 3(-2)^5 - (-2)^3 + 6(-2)^2 - (-2) + 1 = -61$

$\boxed{-61 \text{ is remainder}}$ when $f(x)$ is ÷ by $(x+2)$.

3. Find the zeros for the function $f(x) = x^3 + 3x^2 - x - 3$

$x^2(x+3) - 1(x+3)$

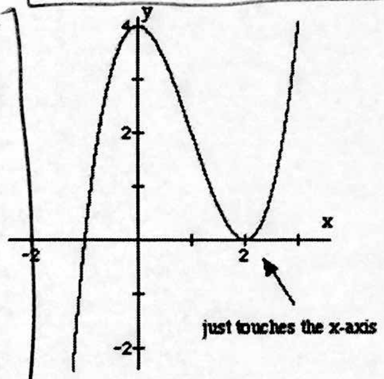
$(x^2-1)(x+3)$ → $(x+1)(x-1)(x+3)$

$\boxed{x = -1, 1, -3}$

4. Show whether -4 is a zero of $g(x) = x^3 - x^2 - 14x + 24$

$g(-4) = (-4)^3 - (-4)^2 - 14(-4) + 24 = 8$ $\boxed{-4 \text{ is } \underline{\text{not}} \text{ a zero of } g(x)}$

5. Use the graph to answer the following questions



just touches the x-axis

a) ~~Relative~~ maximum: 4 $(0,4)$

b) Relative minimum: 0 $(2,0)$

c) Increasing interval: $(-\infty,0) \cup (2,\infty)$

d) Decreasing interval: $(0,2)$

e) Domain: $(-\infty,\infty)$

f) Range: $(-\infty,\infty)$

g) End Behavior: $x \to -\infty\ f(x) \to -\infty$ $x \to \infty\ f(x) \to \infty$

h) Zeros: $x = -1$ $x = 2$ mult. 2

i) Factored Form: $y = a(x+1)(x-2)^2$

j) Degree 3

Find all the zeros

6. $f(x) = 2x^3 + 3x^2 - 39x - 20$ and 4 is a zero

7. $f(x) = x^4 + 3x^2 - 4$ and 1 is a zero

7-9 on back!

⑥ $x-4 \overline{\big) 2x^3+3x^2-39x-20}$ quotient $2x^2+11x+5$

$-(2x^3-8x^2)$

$11x^2-39x$

$-(11x^2-44x)$

$5x-20$

Factor $2x^2+11x+5$

$(2x+1)(x+5)=0$

$\boxed{x = -1/2 \quad x = -5}$ Also $x=4$

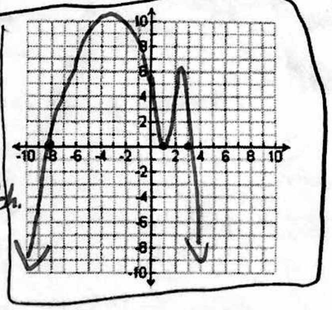
Divide using long division

8. $x^3 - 3x^2 + 8x - 5 \div (x - 1)$

9. $4x^3 - 12x^2 - x + 15 \div (2x - 3)$

10. Sketch a graph $f(x) = -4(x - 1)^2(x - 3)(x + 8)$

use calc if you want, but I don't. I use zeros w/ mult. + End. behav. to sketch.



11. Write the function of x^4 shifted 3 units down, 4 units left, a reflection over the x-axis and a horizontal compression by 3

$\boxed{y = -\left(\frac{x}{3}+4\right)^4 - 3}$

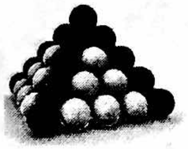
12. A cement walk of uniform width surrounds a rectangular swimming pool that is 10 m wide and 50 m long. Find the width of the walk if its area is 896 m².

$(10+2x)(50+2x)=896$ Use calc. $\boxed{x = 3m}$

13. The number of eggs, f(x), in a female moth is a function of her abdominal width, x, in millimeters, modeled by $f(x) = 14x^3 - 17x^2 - 16x + 34$. What is the abdominal width when there are 211 eggs? $211 =$

use calc. so $\boxed{x = 3\ mm}$

14. A pyramid can be formed using ~~equal-size balls. For~~ example, 3 balls can be arranged in a triangle, then a fourth ball placed in the middle on top of them. The function $p(n) = \frac{1}{6}n(n+1)(n+2)$ gives the number of balls in a pyramid, where n is the number of balls on each side of the bottom layer. (For the pyramid described above, $n = 2$. For the pyramid in the picture, $n = 5$.)



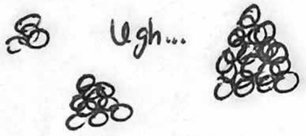
a. Evaluate $p(2)$, $p(3)$, and $p(4)$. Sketch a picture of the pyramid that goes with each of these values. Check that your function values agree with your pyramid pictures.

b. If you had 1000 balls available and you wanted to make the largest possible pyramid using them, what would be the size of the bottom triangle, and how many balls would you use to make the pyramid? How many balls would be left over?

a) $p(2) = \frac{1}{6}(2)(2+1)(2+2) = 4$

$p(3) = \frac{1}{6}(3)(3+1)(3+2) = 10$

$p(4) = \frac{1}{6}(4)(4+1)(4+2) = 20$



b) $1000 = \frac{1}{6}(n)(n+1)(n+2)$

intersection is $n = 17.19$. So use $n = 17$.

$f(17) = \frac{1}{6}(17)(17+1)(17+2) = 969$ balls used.

31 left over

⑦

$$x^3+x^2+4x+4$$

$$x-1\,\overline{)\,x^4+0x^3+3x^2+0x-4}$$

$$-(x^4-x^3)$$

$$x^3+3x^2$$

$$-(x^3-x^2)$$

$$4x^2+0x$$

$$-(4x^2-4x)$$

$$4x-4$$

Factor x^3+x^2+4x+4

$$x^2(x+1)+4(x+1)$$

$$(x^2+4)(x+1)$$

$x^2+4=0$ $\quad$ $x+1=0$

$x^2=-4$ $\quad$ $\boxed{x=-1}$

$\boxed{x=\pm 2i}$

Also $x=1$

⑧

$$\boxed{x^2-7x-4-\frac{9}{x-1}}$$

$$x-1\,\overline{)\,x^3-8x^2+3x-5}$$

$$-(x^3-x^2)$$

$$-7x^2+3x$$

$$-(-7x^2+7x)$$

$$-4x-5$$

$$-(-4x+4)$$

$$-9$$

⑨

$$\boxed{2x^2-3x-5}$$

$$2x-3\,\overline{)\,4x^3-12x^2-x+15}$$

$$-(4x^3-6x^2)$$

$$-6x^2-x$$

$$-(-6x^2+9x)$$

$$-10x+15$$

$$-10x+15$$

$$0$$