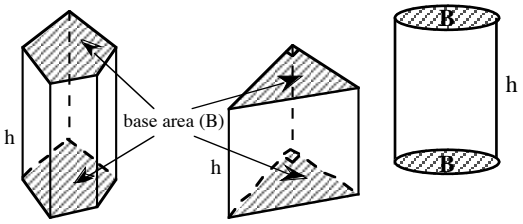
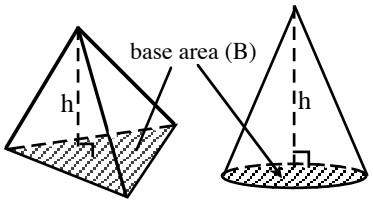
VOLUME AND SURFACE AREA OF POLYHEDRA #17

The **VOLUME** of various polyhedra, that is, the number of cubic units needed to fill each one, is found by using the formulas below.

for prisms and cylinders
$V = \text{base area} \times \text{height}$, $V = Bh$



for pyramids and cones
$V = \frac{1}{3}Bh$



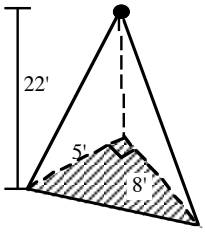
In prisms and cylinders, you may use either base, since they are congruent. Since the bases of cylinders and cones are circles, their area formulas may be expressed as: cylinder $V = \pi r^2 h$ and cone $V = \frac{1}{3}\pi r^2 h$

The **SURFACE AREA** of a polyhedron is the sum of the areas of its base(s) and faces.

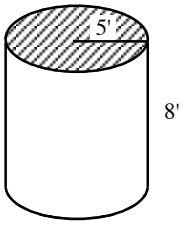
Example 1

Use the appropriate formula(s) to find the volume of each figure below:

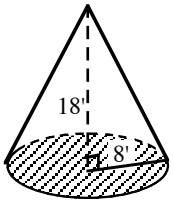
a)



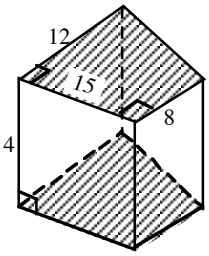
b)



c)



d)



a) This is a triangular pyramid. The base is a right triangle so the area of the base is $B = \frac{1}{2} \cdot 8 \cdot 5 = 20$ square units, so $V = \frac{1}{3}(20)(22) \approx 146.7$ cubic feet.

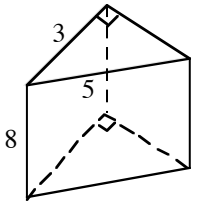
b) This is a cylinder. The base is a circle, so $B = \pi 5^2$, $V = (25\pi)(8) = 200\pi \approx 628.32$ cubic feet.

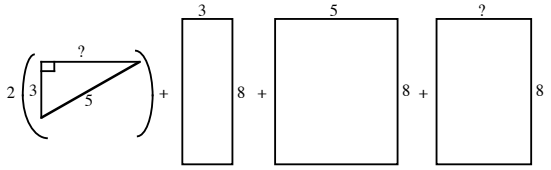
c) This is a cone. The base is a circle, so $B = \pi 8^2$. $V = \frac{1}{3}(64\pi)(18) = \frac{1}{3}(64)(18)\pi$
$\Rightarrow = \frac{1}{3}(1152)\pi = 384\pi \approx 1206.37 \text{ feet}^3$

d) This prism has a trapezoidal base, so $B = \frac{1}{2}(12+8)(15) = 150$.
Thus, $V = (150)(14) = 2100$ cubic feet.

Example 2

Find the surface area of the triangular prism shown at right. The figure is made up of two triangles (the top and bottom) and three rectangles as shown at right. Find the area of each of these shapes.





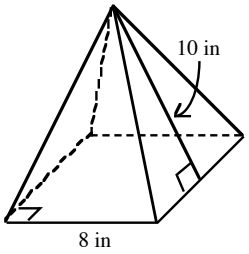
To find the area of the triangle and the last rectangle, use the Pythagorean Theorem to find the length of the second leg of the right triangular base. Since $3^2 + \text{leg}^2 = 5^2$, leg = 4.

Calculate all of the areas, and find their sum.

$$\text{SA} = 2\left(\tfrac{1}{2}(3)(4)\right) + 3(8) + 5(8) + 4(8)$$
$$= 12 + 24 + 40 + 32 = 108 \text{ square units}$$

Example 3

Find the total surface area of a regular square pyramid with a slant height of 10 inches and a base with sides 8 inches long.

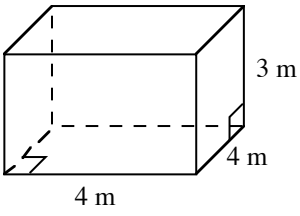


$$\text{SA} = 4\left(\frac{10 \times 8}{2}\right) + 8(8)$$
$$= 160 + 64$$
$$= 224 \text{ square inches}$$

The figure is made up of 4 identical triangles and a square base.

Find the volume of each figure.

1.



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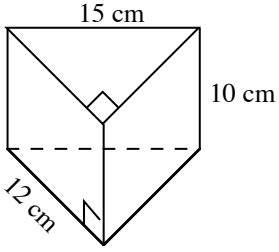

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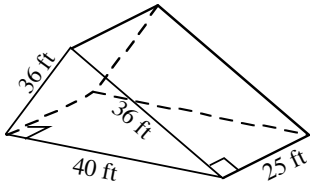

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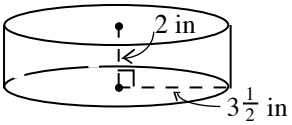

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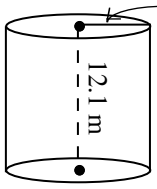

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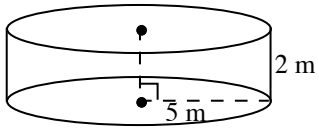
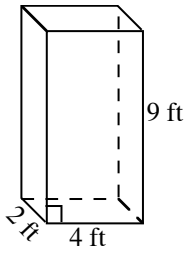
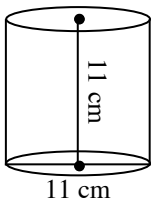
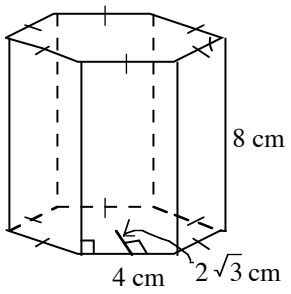
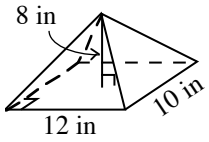
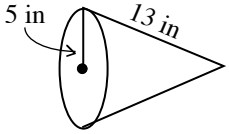


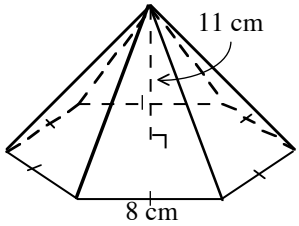
7. 

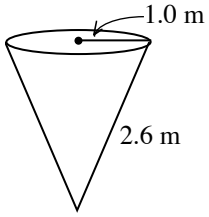
8. 

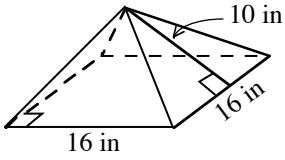
9. 

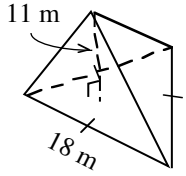
10. 

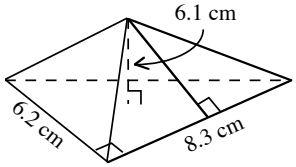
11. 

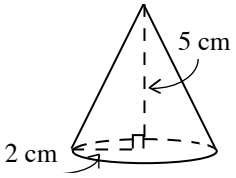
12. 

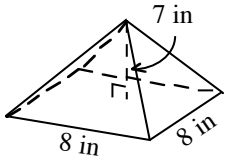
13. 

14. 

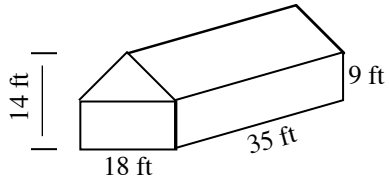
15. 

16. 

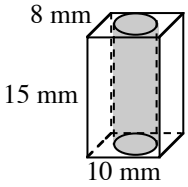
17. 

18. 

19. Find the volume of the solid shown.



20. Find the volume of the remaining solid after a hole with a diameter of 4 mm is drilled through it.



Find the total surface area of the figures in the previous volume problems.

21. Problem 1
22. Problem 2
23. Problem 3
24. Problem 5
25. Problem 6
26. Problem 7
27. Problem 8
28. Problem 9
29. Problem 10
30. Problem 14
31. Problem 18
32. Problem 19

Answers

1. 48 m^3
2. 540 cm^3
3. 14966.6 ft^3
4. 76.9 m^3
5. 1508.75 m^3
6. 157 m^3
7. 72 ft^3
8. 1045.4 cm^3
9. 332.6 cm^3
10. 320 in^3
11. 314.2 in^3
12. 609.7 cm^3
13. 2.5 m^3
14. 512 m^3
15. 514.4 m^3
16. 2.3 cm^3
17. 20.9 cm^3
18. 149.3 in^3
19. 7245 ft^3
20. 1011.6 mm^3
21. 80 m^2
22. 468 cm^2
23. 3997.33 ft^2
24. 727.98 m^2
25. $50\pi + 20\pi \approx 219.8$ m^2
26. 124 ft^2
27. $121\pi + 189.97 \approx 569.91$ cm^2
28. $192 + 48\sqrt{3} \approx 275.14$ cm^2
29. 213.21 in^2
30. 576 in^2
31. 193.0 in^2
32. 2394.69 ft^2