

4.6 Combinations and Compositions

$$f(x) = 2x - 5 \quad g(x) = x^2 - 3x + 6$$

$$1. (f+g)(x) = (2x-5) + (x^2-3x+6) = \boxed{x^2 - x + 1}$$

$$2. (f-g)(x) = (2x-5) - (x^2-3x+6) = \boxed{-x^2 + 5x - 11}$$

$$3. (fg)(x) = (2x-5)(x^2-3x+6)$$
$$2x^3 - 6x^2 + 12x - 5x^2 + 15x - 30$$
$$\boxed{2x^3 - 11x^2 + 27x - 30}$$

$$4. \left(\frac{f}{g}\right)(x) = \frac{2x-5}{x^2-3x+6}$$

$$5. (f+g)(2) \quad (f+g)(x) = x^2 - x - 1$$
$$(f+g)(2) = (2)^2 - 2 - 1 = \boxed{1}$$

$$6. (f-g)(3) \quad (f-g)(x) = -x^2 + 5x - 11$$
$$(f-g)(3) = -(3)^2 + 5(3) - 11 = -9 + 15 - 11 = \boxed{-5}$$

$$7. (fg)(-2) \quad (fg)(x) = 2x^3 - 11x^2 + 27x - 30$$
$$(fg)(-2) = 2(-2)^3 - 11(-2)^2 + 27(-2) - 30 = \boxed{-4}$$

$$8. \left(\frac{f}{g}\right)(6) \quad \left(\frac{f}{g}\right)(x) = \frac{2x-5}{x^2-3x+6} \quad \left(\frac{f}{g}\right)(6) = \frac{2(6)-5}{(6)^2-3(6)+6} = \frac{7}{24}$$

$$9. (f \circ g)(1) = f(g(1)) \quad g(1) = 1^2 - 3(1) + 6 = 4$$
$$f(4) = 2(4) - 5 = \boxed{3}$$

$$10. (g \circ f)(-2) = g(f(-2)) \quad f(-2) = 2(-2) - 5 = -9$$
$$g(-9) = (-9)^2 - 3(-9) + 6 = \boxed{114}$$

$$11. (f \circ g)(3) \quad g(3) = 2(3) - 3 = 3$$
$$f(3) = 2(3)^2 - 5(3) + 1 = \boxed{4}$$

$$12. (g \circ f)(1) \quad f(1) = 2(1)^2 - 5(1) + 1 = -2$$
$$g(-2) = 2(-2) - 3 = \boxed{-7}$$

$$13. (f+g)(-4) = (-4)^2 + 4 + 3(4) + 6 = \boxed{38}$$

$$14. \left(\frac{f}{g}\right)(12) = \frac{(12)^2 + 4}{3(12) + 6} = \frac{148}{42} = \boxed{\frac{74}{21}}$$

$$15. (f \circ g)(x) = (3x+6)^2 + 4$$

$$\text{or } (3x+6)(3x+6) + 4$$

$$9x^2 + 18x + 18x + 36 + 4 = \boxed{9x^2 + 36x + 40}$$

$$16. (g \circ f)(x) = 3(x^2 + 4) + 6$$

$$3x^2 + 12x + 6 = \boxed{3x^2 + 18}$$

$$17. \left(\frac{f}{g}\right)(x) = \frac{x^2 + 4}{3x + 6}$$

$$x \neq -2$$

$$\boxed{(-\infty, -2) \cup (-2, \infty)}$$

$$18. f(x) = |x+2| \quad g(x) = -x$$

$$f(g(1)) = g(1) = -1$$

$$f(-1) = |-1+2| = |1| = \boxed{1}$$

$$g(f(3)) \quad f(3) = |3+2| = 5$$

$$g(5) = \boxed{-5}$$

$$19. f(f(0)) = \boxed{0}$$

$$19. f(x) = x \quad g(x) = -3$$

$$A. f(g(1)) = g(1) = -3 \quad f(-3) = \boxed{-3}$$

$$B. g(f(3)) \quad f(3) = 3 \quad g(3) = \boxed{-3}$$

$$C. f(f(0)) \quad f(0) = 0 \quad f(0) = 0$$

$$20. f(x) = x^2 - 1 \quad g(x) = \sqrt{x}$$

$$A. f(g(1)) \quad g(1) = \sqrt{1} = 1 \quad f(1) = 1^2 - 1 = \boxed{0}$$

$$B. g(f(3)) \quad f(3) = 3^2 - 1 = 8 \quad g(8) = \sqrt{8} \text{ or } \boxed{2\sqrt{2}}$$

$$C. f(g(0)) \quad f(0) = 0^2 - 1 = -1 \quad f(-1) = (-1)^2 - 1 = \boxed{0}$$