Midterm Review Key

1. $-\frac{5\pi}{6} + 2\pi = \frac{-5\pi}{6} + \frac{12\pi}{6} = \boxed{\frac{7\pi}{6}}$

2. $\cos\left(\frac{-3\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$
   same as $\cos(5\pi/4)$

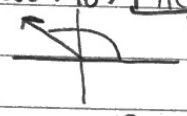
3. $\tan\frac{2\pi}{8} = \boxed{-\sqrt{3}}$

4. $\sec\theta < 0 \quad \cot\theta < 0$

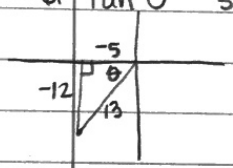
S	A
T	C

$\boxed{QII}$

5. $\cos\left(\frac{9\pi}{10}\right)$ $\boxed{\text{negative}}$



6. $\tan\theta = \frac{12}{5}$ QIII

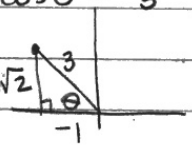


$\sin\theta = \frac{-12}{13} \quad \csc\theta = \frac{-13}{12}$

$\cos\theta = \frac{-5}{13} \quad \sec\theta = \frac{-13}{5}$

$\cot\theta = \frac{5}{12}$

7. $\cos\theta = \frac{-1}{3} \quad \pi/2 < \theta < \pi$



$\csc\theta = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{3\sqrt{2}}{4}}$

$(-1)^2 + y^2 = 3^2$

$1 + y^2 = 9$

$y^2 = 8$

$y = \sqrt{8} = 2\sqrt{2}$

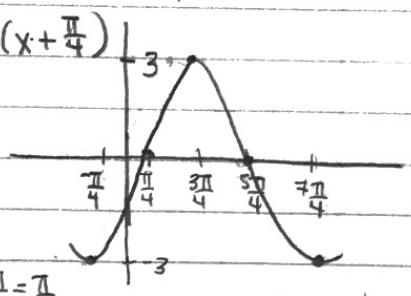
8. $\boxed{y = \frac{1}{2}(\pi x) - 4}$

$\frac{2\pi}{b}$ = period = 2, $b = \pi$

9. $y = \sin\left(\frac{\pi}{4}x\right) \quad P = \frac{2\pi}{\pi/4} = 2\pi \cdot \frac{4}{\pi} = \boxed{8}$

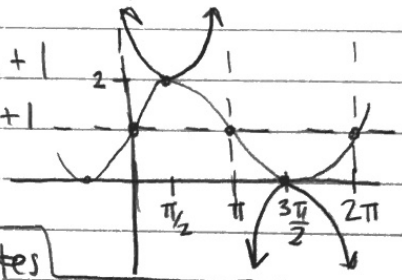
10. $y = -3\cos\left(x + \frac{\pi}{4}\right)$

amp 3
period 2π
reflection
phase shift $-\frac{\pi}{4}$
key points: $\frac{2\pi}{4} = \frac{\pi}{2}$



max occurs at $\boxed{x = \frac{3\pi}{4}}$

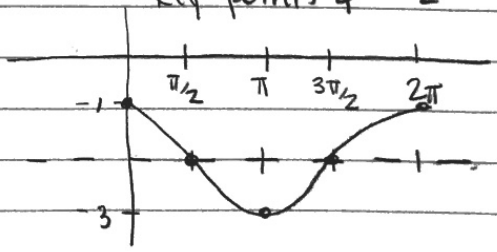
11. $y = \csc x + 1$
    $y = \sin x + 1$



asymptotes occur at $x = 0, \pi, 2\pi$

12. $y = \cos\left(x - \frac{\pi}{2}\right) - 2$

amp = 1
period = 2π
shift down 2
phase shift right $\pi/2$
key points $\frac{2\pi}{4} = \pi/2$



13. $\frac{\tan\alpha - \cos\alpha}{\cot\alpha \quad \sec\alpha}$

$= \frac{\frac{\sin\alpha}{\cos\alpha} - \cos\alpha}{\frac{\cos\alpha}{\sin\alpha} \quad \frac{1}{\cos\alpha}}$

$= \frac{\sin\alpha \cdot \sin\alpha - \cos\alpha \cdot \cos\alpha}{\cos\alpha - \cos\alpha}$

$= \frac{\sin^2\alpha - \cos^2\alpha}{\cos^2\alpha}$

$= \boxed{\tan^2\alpha - \cos^2\alpha}$

14. $\frac{\cot x}{\cos x} + \frac{1}{\sin x}$

$\frac{\frac{\cos x}{\sin x}}{\cos x} + \frac{1}{\sin x}$

$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} + \frac{1}{\sin x}$

$\frac{1}{\sin x} + \frac{1}{\sin x}$

$\frac{2}{\sin x} = \boxed{2\csc x}$

15. $2\csc^2 x = \frac{1}{1-\cos x} + \frac{1}{1+\cos x}$

$\frac{(1+\cos x)}{(1+\cos x)} \quad \frac{(1-\cos x)}{(1-\cos x)}$

$= \frac{1+\cos x + 1-\cos x}{(1-\cos x)(1+\cos x)}$

$= \frac{2}{1-\cos^2 x}$

$= \frac{2}{\sin^2 x} = \boxed{2\csc^2 x}$

16. $2\sin\theta = -\sqrt{2}$

$\sin\theta = \frac{-\sqrt{2}}{2}$

$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

17. $4\cos^2\theta = 4$

$\cos^2\theta = 1$

$\cos\theta = \pm 1$

$\theta = 0$ or $2\pi, \pi$

18. $\sec x - \sqrt{2} = 0$

$\sec x = \sqrt{2}$

$\frac{1}{\cos x} = \frac{\sqrt{2}}{1}$

$\cos x = \frac{1}{\sqrt{2}}$

$\cos x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

19. $2\sin^2 x - \sin x - 1 = 0$

* think $2x^2 - x - 1$

$(2\sin x + 1)(\sin x - 1) = 0 \quad (2x+1)(x-1)$

$2\sin x + 1 = 0 \quad \sin x - 1 = 0$

$\sin x = \frac{-1}{2} \quad \sin x = 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

20. $\sin 2\theta = -\cos\theta$

$2\sin\theta\cos\theta = -\cos\theta$

$2\sin\theta\cos\theta + \cos\theta = 0$

$\cos\theta(2\sin\theta + 1) = 0$

$\cos\theta = 0 \quad 2\sin\theta + 1 = 0$

$\sin\theta = \frac{-1}{2}$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

21. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6} - \sqrt{2}}{4}$

28. $f(x) = x^3$

$f(x) = \left[\frac{1}{4}(x+3)\right]^3 + 1$

29. a. $f(x) = x^2+1$ $\mathbb{R}$ $(-\infty, \infty)$

b. $f(x) = \sqrt{x+2}$ $x+2 \geq 0$

$x \geq -2$

$[-2, \infty)$

c. $f(x) = \frac{x}{2x-1}$ $2x-1 \neq 0$

$x \neq \frac{1}{2}$

$(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

d. $f(x) = \frac{\sqrt{x}}{x-4}$ $x \geq 0$ $x-4 \neq 0$

$x \geq 0$ $x \neq 4$

$[0, 4) \cup (4, \infty)$

30. $f(x) = x-2$ $g(x) = x^2+3$ $h(x) = \sqrt{x}$

a. $(f+g)(x) = (x^2-2)+(x^2+3)$

$= 2x^2+1$ D: $(-\infty, \infty)$

b. $(f-g)(x) = (x^2-2)-(x^2+3)$

$= -5$ D: $(-\infty, \infty)$

c. $(fg)(x) = (x-2)(x^2+3)$

$= x^3+3x-2x^2-6$

$= x^3-2x^2+3x-6$ D: $(-\infty, \infty)$

d. $\left(\frac{f}{g}\right)(x) = \frac{x-2}{x^2+3}$ D: $(-\infty, \infty)$

e. $(f \circ g)(x) = f(g(x))$

$= (x^2+3)-2$

$= x^2+1$ D: $(-\infty, \infty)$

f. $(g \circ f)(x) = (x-2)^2+3$

$= (x-2)(x-2)+3$

$= x^2-4x+4+3$

$= x^2-4x+7$ D: $(-\infty, \infty)$

g. $(h \circ f)(x) = \sqrt{x-2}$ $x-2 \geq 0$

D: $[2, \infty)$ $x \geq 2$

h. $(f \circ g)(9)$

$g(9) = 9^2+3 = 84$

$f(84) = 84-2 = 82$

31. a. $f(x) = 3x^6 - 5x^4$

$f(-x) = 3(-x)^6 - 5(-x)^4$

$f(-x) = 3x^6 - 5x^4$

$f(-x) = f(x)$ EVEN

b. $f(x) = x^2+2$

$f(-x) = (-x)^2+2$

$f(-x) = x^2+2$

$f(-x) = f(x)$ EVEN

c. $f(x) = x^{101} + 11x$

$f(-x) = (-x)^{101} + 11(-x)$

$f(-x) = -x^{101} - 11x$

$f(-x) = -1(x^{101} + 11x)$

$f(-x) = -1 \cdot f(x)$ ODD

32. a. $f(x) = 2x^2-1$

- does not pass HLT
- no inverse unless we restrict domain.

If $f(x) = 2x^2-1$, $x \geq 0$

$y = 2x^2 - 1$

$x = 2y^2 - 1$

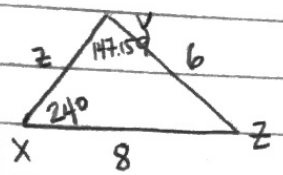
$x+1 = 2y^2$

$y^2 = \frac{1}{2}(x+1)$

$y = \sqrt{\frac{1}{2}(x+1)}$

$f^{-1}(x) = \sqrt{\frac{1}{2}(x+1)}$

22.



$\frac{\sin Y}{8} = \frac{\sin 24}{6}$

$\sin Y = \frac{8 \sin 24}{6}$

$Y = \sin^{-1}\left(\frac{8 \sin 24}{6}\right)$

$Y = 32.841°$ → not obtuse

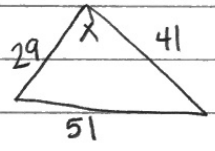
$Y = 180 - 32.841 = 147.159°$

$m\angle Z = 8.841°$

$\frac{Z}{\sin 8.841} = \frac{6}{\sin 24}$

$Z = \frac{6 \sin 8.841}{\sin 24} = \boxed{2.267}$

23.



$51^2 = 29^2 + 41^2 - 2 \cdot 29 \cdot 41 \cos x$

$2601 = 2522 - 2378 \cos x$

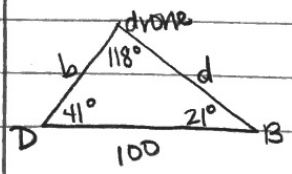
$79 = -2378 \cos x$

$\frac{-79}{2378} = \cos x$

$x = \cos^{-1}\left(\frac{-79}{2378}\right)$

$\boxed{x = 91.90°}$

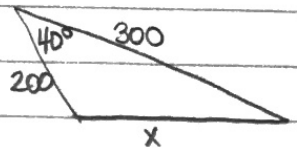
24.



$\frac{b}{\sin 21} = \frac{100}{\sin 108}$ $\quad \frac{d}{\sin 41} = \frac{100}{\sin 108}$

$b = \frac{100 \sin 21}{\sin 108} = \boxed{37.68}$ $\quad d = \frac{100 \sin 41}{\sin 108} = \boxed{68.98}$

25.

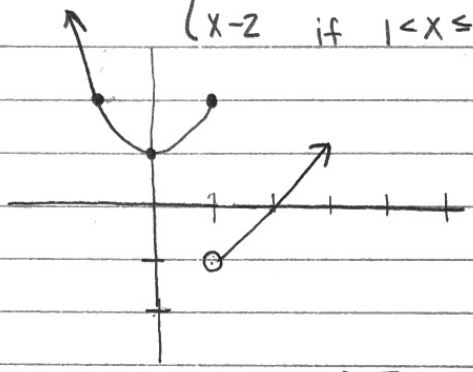


$x^2 = 200^2 + 300^2 - 2(200)(300)\cos 40$

$x^2 = 38074.66083$

$\boxed{x = 195.13}$

26. $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ x - 2 & \text{if } 1 < x \leq 4 \end{cases}$



a. $f(-1) = (-1)^2 + 1 = \boxed{2}$

b. $f(2) = 2 - 2 = \boxed{0}$

c. $f(1) = (1)^2 + 1 = \boxed{2}$

d. $f(4) = 4 - 2 = \boxed{2}$

27. a. $g(x) = \frac{1}{2}\sqrt{x} - 1$

- vertical compression of $\frac{1}{2}$
- down 1

b. $g(x) = -\sqrt{x - 2}$

- reflection about x-axis
- right 2

c. $g(x) = 2\sqrt{3x + 12}$

$g(x) = 2\sqrt{3(x + 4)}$

- vertical stretch of 2
- horizontal compression of $\frac{1}{3}$
- right 4

32. b. $f(x)=\sqrt{3x+4}$ D:$[-\frac{4}{3},\infty)$

$y=\sqrt{3x+4}$ R:$[0,\infty)$

$x=\sqrt{3y+4}$

$x^2=3y+4$

$x^2-4=3y$

$\boxed{y=\frac{1}{3}(x^2-4),\ D:[0,\infty)}$

c. $f(x)=x-1$

$y=x-1$

$x=y-1$

$x+1=y$

$\boxed{f^{-1}(x)=x+1}$

33. a. $f(x)=x^5$ $g(x)=\sqrt[5]{x}$

$f(g(x))=(\sqrt[5]{x})^5=x$

$g(f(x))=\sqrt[5]{x^5}=x$

$\boxed{\text{yes}}$

b. $f(x)=x^3-1$ $g(x)=\sqrt[3]{x}-1$

$f(g(x))=(\sqrt[3]{x}-1)^3+1 \neq x$

$\boxed{\text{no}}$

34. $h(x)=\frac{1}{(x-2)^2}$

$\boxed{\begin{array}{l}h(x)=f(g(x))\\ \text{if } g(x)=x-2\\ \quad f(x)=\frac{1}{x^2}\end{array}}$

35. a. $f(0)=3$ $f(-6)=-3$

b. $f(2)>0$ positive

c. $x=-3,5$

d. 3

e. $f(f(5))$ $f(5)=0$

$f(0)=\boxed{3}$

f. D:$[-6,8]$

g. R:$[-3,4]$

h. $f(x)>0$ on $(-3,5)$

i. $f(x)$ is increasing on $(-6,2)\cup(7,8)$

j. $f(x)$ is decreasing on $(2,7)$