Pre-Calculus Graphing Polynomials Notes **Name** ____________________

Polynomial Function: Let n be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$, be real numbers with $a_n \neq 0$. The function defined by $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ is called a polynomial. The number a_n, the coefficient of the variable to the highest power, is called the leading coefficient.

Ex. $f(x) = 4x^5 - 3x^2 + 2x - 1$ has a degree of 5. The leading coefficient is 4.

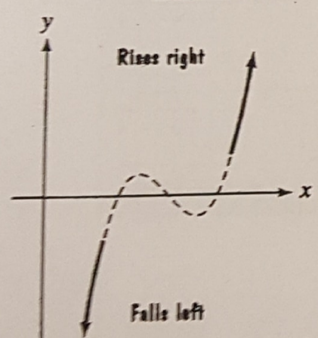
Note: The graphs of all polynomial functions are both smooth and continuous.

- Smooth - no points (like absolute value)
- Continuous - can draw w/out picking up pencil

The Leading Coefficient Test – determines the end behavior of a function.

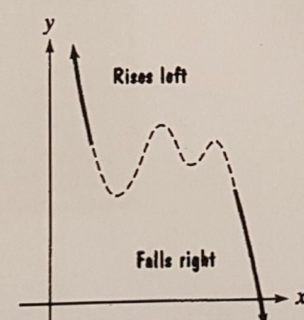
As x increases or decreases without bound, the graph of the polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_0$ $(a_n \neq 0)$ eventually rises or falls. In particular,

1. For *n* odd:
   a. If $a_n > 0$, the graph falls to the left and rises to the right.



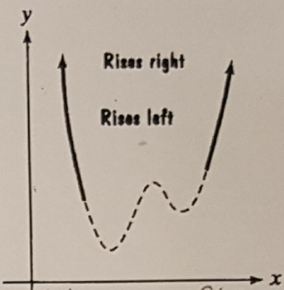
$f(x) \to \infty$ as $x \to \infty$ $f(x) \to -\infty$ as $x \to -\infty$

   b. If $a_n < 0$, the graph falls to the right and rises to the left.



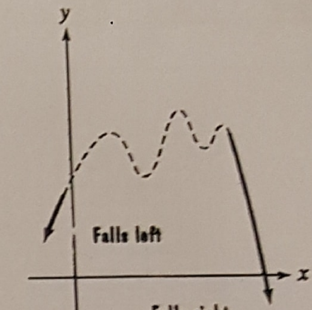
$f(x) \to -\infty$ as $x \to \infty$ $f(x) \to \infty$ as $x \to -\infty$

2. For *n* even:
   a. If $a_n > 0$, the graph rises to the left and to the right.



$f(x) \to \infty$ as $x \to -\infty$ $f(x) \to \infty$ as $x \to \infty$

   b. If $a_n < 0$, the graph falls to the left and to the right.



$f(x) \to -\infty$ as $x \to -\infty$ $f(x) \to -\infty$ as $x \to \infty$

Ex. Determine the end behavior of $f(x) = x^3 + 3x^2 - x - 3$.

odd, $a_n > 0$

$f(x) \to \infty$ as $x \to \infty$ $f(x) \to -\infty$ as $x \to -\infty$

Zeros and Multiplicity: To determine where a function crosses the x-axis, identify its zeros by setting the function $= 0$ and factor.

- If r is a zero of even multiplicity, then the graph bounces off the x-axis and turns around at r.
- If r is a zero of odd multiplicity, then the graph crosses through the x-axis at r.
- **Regardless of whether a zero is even or odd, graphs tend to flatten out at zeros with multiplicity greater than one.

Ex. Find the zeros of $f(x) = -x^4 + 4x^3 - 4x^2$. Determine their multiplicity and describe the graph at each zero.

$-x^2(x^2 - 4x + 4) = 0$

$-x^2(x-2)(x-2) = 0$

$-x^2(x-2)^2 = 0$

$x = 0$ — mult 2 — bounces off x-axis

$x = 2$ — mult 2 — bounces off x-axis

To Sketch a Polynomial:

a) Use the Leading Coefficient Test to determine the polynomial function's end behavior.

b) Find the x-intercepts by setting the function =0 and factoring.

c) Determine each solution's multiplicity and state if it touches the x-axis and turns around or crosses the x-axis.

d) Determine the y-intercept of each polynomial function.

e) Sketch the graph of each polynomial function.

Ex 1) $f(x) = x^3 - 5x^2 - 4x + 20$

$x^3 - 5x^2 - 4x + 20 = 0$

$x^2(x-5) - 4(x-5) = 0$

$(x-5)(x^2-4) = 0$

$x = 5 \quad (x-2)(x+2)$

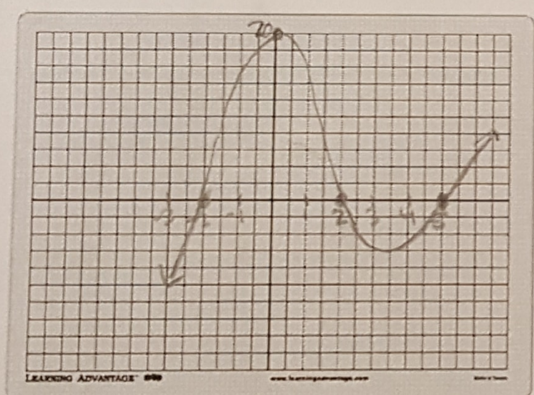
$x = 2 \quad x = -2$

a) $f(x) \to \infty$ as $x \to \infty$ $\quad$ $f(x) \to -\infty$ as $x \to -\infty$

b) $x = 5$, $x = 2$, $x = -2$

c) mult 1 for all (crosses x-axis)

d) 20



Ex 2) $f(x) = -x^4 + 16x^2$

$-x^2(x^2-16) = 0$

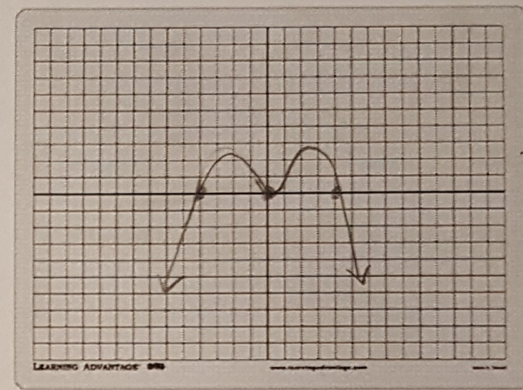
$-x^2(x-4)(x+4) = 0$

a) $f(x) \to -\infty$ as $x \to -\infty$ $\quad$ $f(x) \to -\infty$ as $x \to \infty$

b) $x = 0$ $\quad$ $x = 4$ $\quad$ $x = -4$

c) mult 2 (bounce) $\quad$ mult 1 (crosses) $\quad$ mult 1

d) 0



Ex 3) $f(x) = x^3 + 4x^2 + 4x$

$x(x^2 + 4x + 4)$

$x(x+2)(x+2)$

a) $f(x) \to \infty$ as $x \to \infty$ $\quad$ $f(x) \to -\infty$ as $x \to -\infty$

b) $x = 0$ $\quad$ $x = -2$

c) mult 1 (crosses) $\quad$ mult 2 (bounces)

d) 0