

Multiplying and Dividing Rational Expressions

Example 1: Simplify the following.

$$a) \frac{(x+1)(x-5)}{(x-5)(x^2-1)} = \frac{(x+1)}{(x-1)(x+1)} = \frac{1}{x-1}$$

$x \neq 5, 1, -1$

$$b) \frac{x^2+x-12}{x^2+7x+12} = \frac{(x+4)(x-3)}{(x+4)(x+3)} = \frac{x-3}{x+3}$$

$x \neq -4, -3$

You Try! Simplify the following.

$$a) \frac{x^2+6x+9}{x^2-9} = \frac{(x+3)(x+3)}{(x-3)(x+3)} = \frac{x+3}{x-3}$$

$x \neq \pm 3$

$$b) \frac{4x^2+8x}{x^2+6x+8} = \frac{4x(x+2)}{(x+4)(x+2)} = \frac{4x}{x+4}$$

$x \neq -4, -2$

Multiplying Rational Functions

When multiplying rational functions, meaning you are multiplying two fractions together, you multiply straight across the top and straight across the bottom, simplifying where you can.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example 2: Simplify completely.

$$\frac{x^2+2x-8}{x^2+4x+3} \cdot \frac{3x+3}{x-2}$$

$$\frac{(x+4)(x-2)}{(x+3)(x+1)} \cdot \frac{3(x+1)}{x-2}$$

$$\frac{3(x+4)}{(x+3)} \quad x \neq -3, -1$$

You Try! Multiply the following and state the restrictions.

$$a) \frac{t^2+19t+84}{4t-4} \cdot \frac{2t-2}{t^2+9t+14}$$

$$\frac{(t+12)(t+7)}{4(t-1)} \cdot \frac{2(t-1)}{(t+7)(t+2)}$$

$$\frac{2(t+12)}{4(t+2)} = \frac{t+12}{2(t+2)}$$

$t \neq 1, -7, -2$

Example 3: Simplify Completely.

$$\frac{x^2-9}{x^2+5x+6} \cdot \frac{x+2}{3x-9}$$

$$\frac{(x-3)(x+3)}{(x+5)(x+2)} \cdot \frac{(x+2)}{3(x-3)}$$

$$\frac{1}{3} \quad x \neq -3, -2, 3$$

$$b) \frac{x^2+x-6}{x-5} \cdot \frac{x^2-25}{x^2+4x+3}$$

$$\frac{(x-2)(x+3)}{(x-5)} \cdot \frac{(x-5)(x+5)}{(x+3)(x+1)}$$

$$\frac{(x-2)(x+5)}{x+1} \quad x \neq 5, -3, -1$$

Dividing Rational Functions

When dividing rational functions, you multiply the first fraction by the reciprocal of the second fraction, simplifying where you can. **SAME-CHANGE-FLIP!**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Example 1: Simplify completely and state the restrictions.

$$\frac{a+2}{a+3} \div \frac{a^2+a-12}{a^2-9} = \frac{a+2}{a+3} \cdot \frac{(a-3)(a+3)}{(a-3)(a+4)} = \frac{a+2}{a+4}$$

$a \neq -3, 3, -4$

Example 2: Simplify Completely. State all restrictions.

$$\frac{b^2}{25a^2-b^2} \cdot \frac{5a-b}{5a-b}$$

$$\frac{b^2}{(5a-b)(5a+b)} \cdot \frac{5a-b}{5a-b} = \frac{b^2}{(5a+b)}$$

$b \neq 0$, $5a+b \neq 0$, $a \neq -\frac{b}{5}$

You Try! Divide the following. Be sure to state all restrictions.

$$a) \frac{-12b+18}{b^2-25} \div \frac{4b-6}{b^2-3b-10}$$

$$\frac{-12b+18}{b^2-25} \cdot \frac{b^2-3b-10}{4b-6}$$

$$\frac{-3(4b-6)}{(b-5)(b+5)} \cdot \frac{(b-5)(b+2)}{2(2b-3)}$$

$$\frac{-3(b+2)}{b+5} \quad b \neq -5, 5, \frac{3}{2}$$

$$b) \frac{3x+12}{2x+4} \div \frac{x^2-16}{x+2}$$

$$\frac{3(x+4)}{2(x+2)} \cdot \frac{x+2}{(x-4)(x+4)}$$

$$\frac{3}{2(x-4)} \quad x \neq -2, 4, -4$$