Solving Rational Equations

How to Solve Rational Equations

1. Find a common denominators on each side of the equation
2. Simplify each side into one term, and then cross multiply.
3. Solve for the variable.
4. Check your answers

Example 1: Solve $\frac{20}{3x-5}=\frac{5}{x-2}$

$20(x-2)=5(3x-5)$
$20x-40=15x-25$
$5x=15$
$\boxed{x=3}$ ✓

Example 2: Solve $\frac{x}{x^2-2}=\frac{-1}{x}$

$x^2=-1(x^2-2)$
$x^2=-x^2+2$
$2x^2=2$
$x^2=1$
$x=\pm\sqrt{1}$
$\boxed{x=\pm 1}$

$\frac{1}{1^2-2}=\frac{-1}{1}$
$\frac{1}{-1}=\frac{-1}{1}$
$-1=-1$ ✓
$\frac{-1}{(-1)^2-2}=\frac{-1}{-1}$
$\frac{-1}{-1}=\frac{-1}{-1}$
$1=1$ ✓

Example 3: Solve $\frac{x-3}{x+5}=\frac{x}{x+2}$

$(x-3)(x+2)=x(x+5)$
$x^2+2x-3x-6=x^2+5x$
$-x-6=5x$
$-6=6x$
$-1=x$

$\frac{-1-3}{-1+5}=\frac{-1}{-1+2}$
$\frac{-4}{4}=\frac{-1}{1}$
$-1=-1$ ✓

Example 4: Solve $\frac{2}{3x}+\frac{1}{6}=\frac{4}{3x}$

$\frac{4}{6x}+\frac{x}{6x}=\frac{4}{3x}$
$\frac{4+x}{6x}=\frac{4}{3x}$
$3x(4+x)=24x$
$12x+3x^2=24x$
$3x^2-12x=0$
$3x(x-4)=0$

$3x=0$ ~~$x=0$~~ extraneous
$x-4=0$ $\boxed{x=4}$ ✓

Example 5: Solve $\frac{1}{x-2}+2=\frac{3x}{x+2}$

$\frac{1}{x-2}+\frac{2(x-2)}{x-2}=\frac{3x}{x+2}$
$\frac{1+2x-4}{x-2}=\frac{3x}{x+2}$ ✓
$(2x-3)(x+2)=3x(x-2)$
$2x^2+4x-3x-6=3x^2-6$
$x-6=x^2-6$
$x=x^2$
$0=x^2-x$
$0=x(x-1)$
$x=0$
$x=1$

Example 6: Solve $\frac{2}{x-3}=\frac{1}{x^2-2x-3}$

$2(x^2-2x-3)=x-3$
$2x^2-4x-6=x-3$
$2x^2-5x-3=0$
$(2x+1\quad -\ )=0$
$x+1=0$ ~~$x=3$~~ extraneous
$\boxed{x=-1/2}$

Example 7: $\frac{2}{x-3}+\frac{0}{x}=\frac{1}{x-3}$

$\frac{2x}{x(x-3)}+\frac{1(x-3)}{x(x-3)}=\frac{x-1}{x-3}$
$\frac{2x+x-3}{x(x-3)}=\frac{x-1}{x-3}$
$(3x-3)(x-3)=x(x-3)(x-1)$
$3x^2-9x-3x+9=x(x^2-x-3x+3)$
$3x^2-12x+9=x^3-4x^2+3x$
$0=x^3-7x^2+15x-9$
$0=(x-3)(x^2-4x+3)$
$(x-3)(x-1)$

~~$x=3$~~ extraneous
$\boxed{x=1}$ ✓