

5.9 Rational Functions Review

Directions 1-2: Use synthetic division to divide.

1. $(3x^3 - 4x^2 - 17x + 6) \div (3x - 1)$

$$\begin{array}{r|rrrr} 1/3 & 3 & -4 & -17 & 6 \\ & \downarrow & 1 & -1 & -6 \\ \hline & 3 & -3 & -18 & 0 \end{array}$$

$$\boxed{3x^2 - 3x - 18}$$

2. $(8p^5 + 32p^4 + 5p + 20) \div (p + 4)$

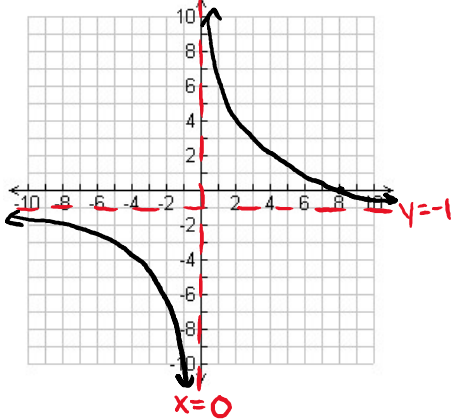
$$\begin{array}{r|rrrrrr} -4 & 8 & 32 & 0 & 0 & 5 & 20 \\ & \downarrow & -32 & 0 & 0 & 0 & -20 \\ \hline & 8 & 0 & 0 & 0 & 5 & 0 \end{array}$$

$$\boxed{8p^4 + 5}$$

Directions: 3- 5: Sketch the asymptotes and the graph of each function. Identify the domain and range.

3. $y = \frac{8}{x} - 1 = \frac{8 - x}{x}$

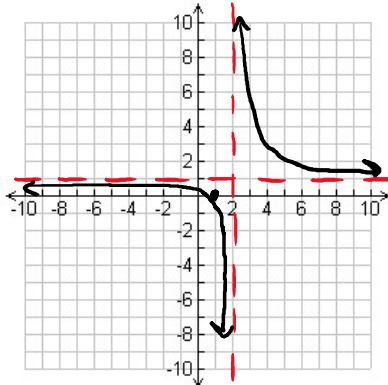
VA: $x = 0$ HA: $y = -1$
x-int: 8



D: $(-\infty, 0) \cup (0, \infty)$
R: $(-\infty, -1) \cup (-1, \infty)$

4. $y = \frac{1}{x-2} + 1 = \frac{x-1}{x-2}$

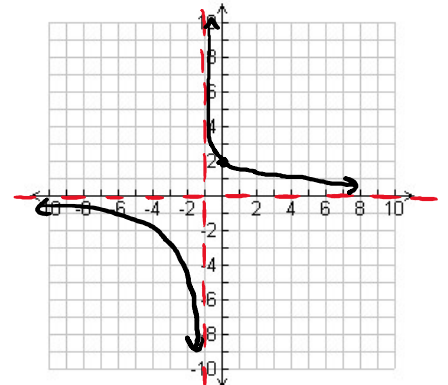
VA: $x = 2$ x-int: 1
HA: $y = 1$



D: $(-\infty, 2) \cup (2, \infty)$
R: $(-\infty, 1) \cup (1, \infty)$

5. $y = \frac{2}{x+1}$

VA: $x = -1$
HA: $y = 0$
x-int: none

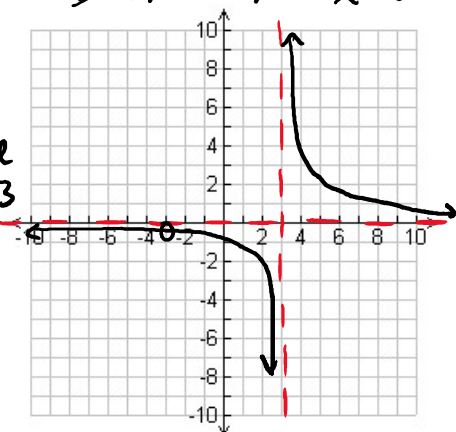


D: $(-\infty, -1) \cup (-1, \infty)$
R: $(-\infty, 0) \cup (0, \infty)$

Directions 6-7: Find points of discontinuity, the domain, and x and y-intercepts of each rational function.

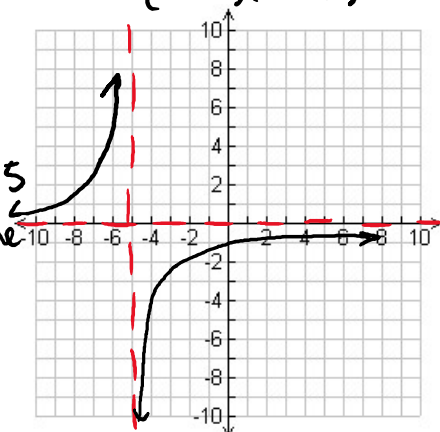
6. $y = \frac{x+3}{x^2-9} = \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{1}{x-3}$

VA: $x = 3$
HA: $y = 0$
x-int: none
Hole: $x = -3$



7. $y = \frac{5-x}{x^2-25} = \frac{-1(x-5)}{(x+5)(x-5)}$

VA: $x = 5$
HA: $y = 0$
Hole: $x = -5$
x-int: none



Directions 8-15: Simplify. State any restrictions on the variables.

$$8. \frac{x^2+9x+18}{x+6} = \frac{(x+6)(x+3)}{(x+6)} = \boxed{x+3}$$

$$\boxed{x \neq -6}$$

$$9. \frac{x^2-2x-8}{x+3} \div \frac{x-4}{x+3}$$

$$\frac{(x-4)(x+2)}{(x+3)} \cdot \frac{(x+3)}{(x-4)} = \boxed{x+2}$$

$$\boxed{x \neq -3, 4}$$

$$10. \frac{2x^2+5x-3}{x^2-4x} \cdot \frac{2x^3-8x^2}{x^2+6x+9}$$

$$\frac{(2x-1)(x+6)}{x(x-4)} \cdot \frac{2x^2(x-4)}{(x+3)(x+3)}$$

$$\boxed{\frac{(2x-1)(x+6)}{(x+3)(x+3)} \quad x \neq 0, 4, -3}$$

$$11. \frac{3x+1}{x^2-x-6} \div \frac{6x^2+11x+3}{x^2+4x+4}$$

$$\frac{(3x+1)}{(x-3)(x+2)} \cdot \frac{(x+2)(x+2)}{(2x+3)(3x+1)}$$

$$\boxed{\frac{(x+2)}{(x-3)(2x+3)} \quad x \neq 3, -2, -\frac{3}{2}, -\frac{1}{3}}$$

$$12. \frac{6x+1}{x+2} + \frac{2x-5}{2x+4}$$

$$\frac{(6x+1)}{x+2} + \frac{2x-5}{2(x+2)} = \frac{2(6x+2)+2x-5}{2(x+2)}$$

$$\frac{12x+4+2x-5}{2(x+2)} = \boxed{\frac{14x-1}{2(x+2)} \quad x = -2}$$

$$13. \frac{3x}{x^2+5x+6} - \frac{2x}{x^2+8x+16}$$

$$\frac{3x}{(x+2)(x+3)} - \frac{2x}{(x+4)(x+4)}$$

$$\frac{3x(x+4)(x+4) - 2x(x+2)(x+3)}{(x+2)(x+3)(x+4)(x+4)}$$

$$\frac{3x(x^2+8x+16) - 2x(x^2+5x+6)}{(x+2)(x+3)(x+4)(x+4)}$$

$$\frac{3x^3+24x^2+48x - 2x^3-10x^2-12x}{(x+2)(x+3)(x+4)(x+4)}$$

$$\frac{x^3+14x^2+36x}{den} = \boxed{\frac{x(x^2+14x+36)}{(x+2)(x+3)(x+4)(x+4)} \quad x \neq -2, -3, -4}$$

$$14. \frac{2}{x^2-1} - 3$$

$$\frac{2}{(x+1)(x-1)} - \frac{3(x+1)(x-1)}{(x+1)(x-1)}$$

$$\frac{2-3x^2+3}{(x+1)(x-1)} = \boxed{\frac{-3x^2+5}{(x+1)(x-1)}}$$

$$15. \frac{x-3}{x^2+3x} + \frac{7}{x+3}$$

$$\frac{x-3}{x(x+3)} + \frac{7}{x+3} = \frac{x-3+7x}{x(x+3)}$$

$$= \boxed{\frac{8x-3}{x(x+3)} \quad x \neq 0, -3}$$

Directions 16-17: Solve each equation. Check each solution.

16. $\frac{3x}{x-2} = 4 + \frac{x}{5}$

$$\frac{3x}{x-2} = \frac{20+x}{5}$$

$$15x = (x-2)(20+x)$$

$$15x = 20x + x^2 - 40 - 2x$$

$$0 = x^2 + 3x - 40$$

$$0 = (x+8)(x-5)$$

$x = -8$ $x = 5$

17. $(x + \frac{x}{4} - \frac{x}{5} = 21) \cdot 20$

$$20x + 5x - 4x = 420$$

$$21x = 420$$

$$x = 20$$

18. It would take an apprentice house painter 1.5 h longer than his supervisor to paint an apartment. If they work together, they can complete the job in 4 h. About how long would it take the apprentice to complete the job working alone? Round your answer to the nearest tenth of an hour.

	Work	Rate	Time
apprentice	1	$\frac{1}{x}$	x
supervisor	1	$\frac{1}{4-x}$	4-x

$$\left(\frac{1}{x} + 1.5 = \frac{1}{4-x}\right) \cdot (2x(4-x))$$

$$2(4-x) + 3x(4-x) = 2x$$

$$8 - 2x + 12x - 3x^2 = 2x$$

$$0 = 3x^2 - 8x - 8$$

quadratic formula

negative answer doesn't work for time

$$x = 3.44 \text{ hours}$$

19. A master roofer can cover a garage in 1 h less than her new assistant. If they work together, they can complete the job in 7.75 h. How long would it take the assistant to complete the job working alone?

	Work	Rate	Time
master	1	$\frac{1}{x-1}$	x-1
assistant	1	$\frac{1}{x}$	x

$$\frac{1}{x-1} + \frac{1}{x} = \frac{1}{7.75}$$

$$\frac{x+1}{x(x-1)} = \frac{1}{7.75}$$

$$7.75x + 7.75 = x^2 - x$$

$$0 = x^2 - 8.75x - 7.75$$

$$x = 9.56$$

$$x = -0.811$$

Assistant can complete the job in 9.56 hours