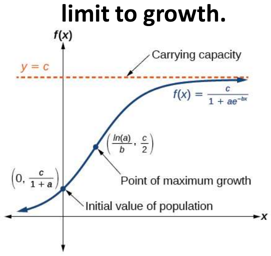
Logistic Growth Modeling

$$f(x)=\frac{c}{1+ae^{-bx}}$$

Where *a, c,* and *b* are positive and c is the **limit to growth.**



Population Growth

The population of New York state can be modeled by

$$P(t)=\frac{19.875}{1+57.993e^{-0.035005t}}$$

where *P* is the population in millions and *t* is the number of years since 1800. Based on the model,

a) What was the population of New York in 1850?

b) What will it be in the year 2010?

c) What is New York's *maximum sustainable population* (limit of growth)?

Logistic Growth

What is the maximum sustainable population given the model?

$$P(t)=\frac{3050}{1+16.912e^{-0.07189t}}$$

Logistic Model

- Use logistic regression [Stat ⇒ Calc ⇒(B)] to predict the maximum sustainable populations for the two countries.
- Will Mexico's population surpass that of the United States? Justify your answer.

Year	US	Mexico
1900	76.2	13.6
1950	92.2	25.8
1960	106.0	34.9
1970	123.2	48.2
1980	226.5	66.8
1990	248.7	88.1
2001	281.4	101.9

Logistic Model

The maximum height, in inches, a ball reaches after its first four bounces is shown in the table below.

Bounce Number	**Height** (in inches)
1	42.0
2	31.5
3	23.6
4	17.7

Which type of function ***best*** models the data and why?

A an exponential function, because the height of the ball is decreasing by 25% with each bounce

B an exponential function, because the height of the ball is decreasing by 75% with each bounce

C a logistic function, because the height of the ball is decreasing by 25% with each bounce

D a logistic function, because the height of the ball is decreasing by 75% with each bounce

Logistic Model

The following equation describes the spread of a rumor around a school of 1200 students. S is the number of students who have heard the rumor by the end of t days (t=0 is the day the rumor starts). How long does it take for 1000 students to hear the rumor?

$$S(t)=\frac{1200}{1+39e^{-0.9t}}$$

Logistic Model

Use the data in the table to compute a logistic regression model for the population of the city t years after 1900. Then use your model to predict the city's population in 1980.

Year	Pop (in millions)
1900	0.8
1910	1.0
1920	1.3
1930	1.7
1940	2.0
1950	2.5
1960	3.0
1970	3.6

Logistic Model

The number of students infected with the flu on a college campus after t days is modeled by the function $P(t) = \frac{440}{1 + 39e^{-0.3t}}$. What is the maximum number of infected students possible?

Logistic Model

The population of wolves in a state park after t years is modeled by the function $P(t) = \frac{900}{1 + 99e^{-0.3t}}$. What was the initial population of wolves?

Logistic Model

Find the logistic function that satisfies the given conditions.

9) Initial value = 10, limit to growth = 40, passing through (1, 16)

10) Initial height = 169, limit to growth = 845, passing through (2, 585)