

7.5 Ellipses

1. $\frac{x^2}{49} + \frac{y^2}{169} = 1$

horizontal major axis

$a^2 = 169$ $b^2 = 49$

$a = 13$ $b = 7$

$c^2 = 169 - 49 = 120$

$c^2 = 120$

$c = 2\sqrt{30} \approx 10.954$

center $(0, 0)$

vertices $(0, 13)$

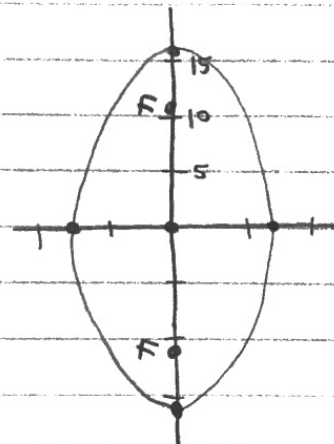
$(0, -13)$

foci $(0, 2\sqrt{30})$

$(0, -2\sqrt{30})$

major axis 26

minor axis 14



2. $\frac{x^2}{64} + \frac{(y-8)^2}{9} = 1$

vertical major axis

$a^2 = 64$ $b^2 = 9$

$a = 8$ $b = 3$

$c^2 = 64 - 9 = 55$

$c = \sqrt{55} \approx 7.416$

center $(0, 8)$

vertices

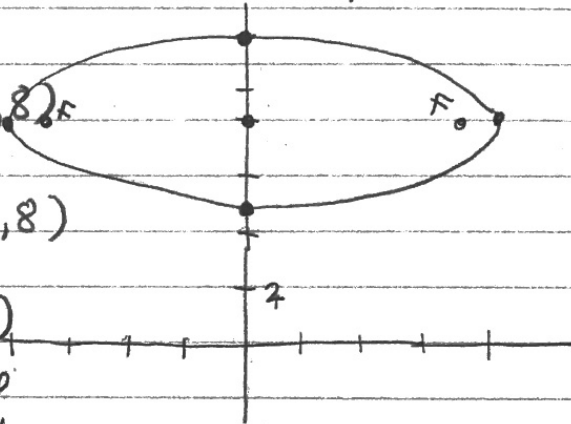
$(8, 8)$ $(-8, 8)$

foci $(\sqrt{55}, 8)$

$(-\sqrt{55}, 8)$

major axis 16

minor axis 6



3. $\frac{(x+3)^2}{12} + \frac{(y-2)^2}{16} = 1$

horizontal major axis

$a^2 = 16$ $b^2 = 12$

$a = 4$ $b = \sqrt{12} = 2\sqrt{3} \approx 3.464$

$c^2 = 16 - 12 = 4$

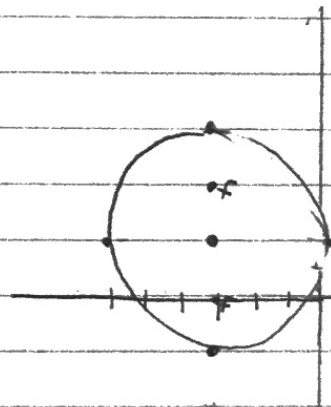
$c = 2$

center $(-3, 2)$

vertices $(-3, 6)$

$(-3, -2)$

foci $(-3, 4)$ $(-3, 0)$



$$4. \frac{x^2}{16} + \frac{16y^2}{16} = \frac{16}{16}$$

$$\frac{x^2}{16} + y^2 = 1$$

vertical major axis

$$a^2 = 16 \quad b^2 = 1$$

$$a = 4 \quad b = 1$$

$$c^2 = 16 - 1 = 15$$

$$c = \sqrt{15} \approx 3.873$$

center (0,0)

vertices (4,0)

(-4,0)

foci ($\sqrt{15}$, 0)

($-\sqrt{15}$, 0)

major axis 8

minor axis 2



$$5. 9x^2 + 4y^2 - 54x + 40y + 37 = 0$$

$$9x^2 - 54x + 4y^2 + 40y = -37$$

$$9(x^2 - 6x + 9) + 4(y^2 + 10y + 25) = -37 + 81 + 100$$

$$\frac{9(x-3)^2}{144} + \frac{4(y+5)^2}{144} = \frac{144}{144}$$

$$\frac{(x-3)^2}{16} + \frac{(y+5)^2}{36} = 1$$

horizontal major axis

$$a^2 = 36 \quad b^2 = 16$$

$$a = 6 \quad b = 4$$

$$c^2 = 36 - 16 = 20$$

$$c = \sqrt{20} = 2\sqrt{5} \approx 4.472$$

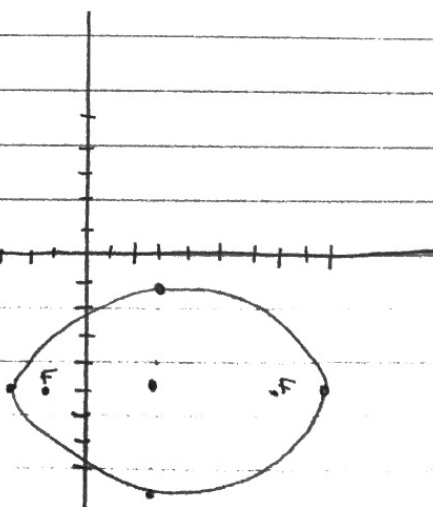
center (3,-5)

vertices (9,-5) (-3,-5)

foci (3+2 $\sqrt{5}$, -5) (3-2 $\sqrt{5}$, -5)

major axis 12

minor axis 8



6. vertices $(10, 0)$ $(-10, 0)$
 co-vertices $(0, 9)$ $(0, -9)$
 $a^2 = 100$
 $b^2 = 81$
 center $(0, 0)$

$$\frac{x^2}{100} + \frac{y^2}{81} = 1$$

9. $x^2 + 4y - 16 = 0$

$$x^2 = -4y + 16$$

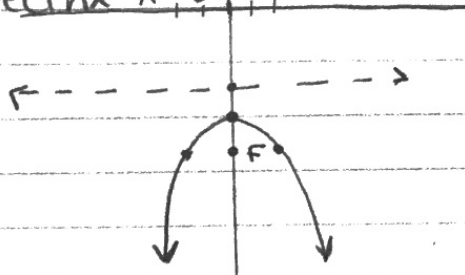
$$x^2 = -4(y - 4)$$

parabola, opens down

vertex $(0, 4)$ $p = 1$

focus $(0, 3)$ $LR = 4$

directrix $x = -3$



7. vertices $(12, 0)$ $(-12, 0)$
 foci $(2\sqrt{11}, 0)$ $(-2\sqrt{11}, 0)$

$$a^2 = 144$$

$$c^2 = (2\sqrt{11})^2 = 4 \cdot 11 = 44$$

$$c^2 = a^2 - b^2$$

$$44 = 144 - b^2$$

$$b^2 = 100$$

$$\frac{x^2}{144} + \frac{y^2}{100} = 1$$

10. $4x^2 + 4y^2 - 20x - 24 = 0$

$$4x^2 - 20x + 4y^2 = 24$$

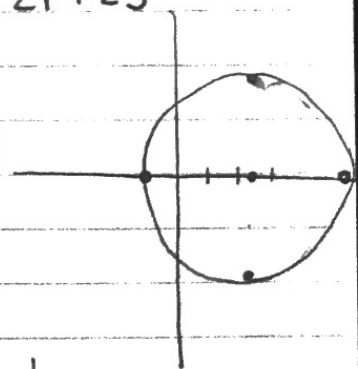
$$4(x^2 - 5x + \frac{25}{4}) + 4y^2 = 24 + 25$$

$$4(x - \frac{5}{2})^2 + 4y^2 = 49$$

$$(x - \frac{5}{2})^2 + y^2 = \frac{49}{4}$$

circle center $(\frac{5}{2}, 0)$

radius $\frac{7}{2}$



8. $x^2 + 4y^2 - 2x - 16y + 1 = 0$

$$x^2 + 2x + 4y^2 - 16y = -1$$

$$x^2 + 2x + 1 + 4(y^2 - 4y + 4) = -1 + 1 + 16$$

$$\frac{(x+1)^2}{16} + \frac{(y-2)^2}{4} = 1$$

$$\frac{(x+1)^2}{16} + \frac{(y-2)^2}{4} = 1$$

ellipse center $(-1, 2)$ vertices $(-5, 2)$ $(3, 2)$

horizontal major axis foci $(-1 + 2\sqrt{3}, 2)$

$a^2 = 16$ $b^2 = 4$ $(-1 - 2\sqrt{3}, 2)$

$$a = 4$$

$$b = 2$$

major axis 8

$$c^2 = 16 - 4 = 12$$

minor axis 4

$$c = \sqrt{12} = 2\sqrt{3}$$

