8.3 Converting Polar and Rectangular Equations

1. $r = 8$
$r^2 = x^2 + y^2$
$8^2 = x^2 + y^2$
$\boxed{64 = x^2 + y^2}$

2. $r\cos\theta = 6$
$x = r\cos\theta$
$\boxed{x = 6}$

3. $r = -5\csc\theta$
$r = \frac{-5}{\sin\theta}$
$r\sin\theta = -5$
$y = r\sin\theta$
$\boxed{y = -5}$

4. $r = 7\sin\theta$
$\frac{r}{7} = \sin\theta$
$y = r\sin\theta$
$\frac{y}{r} = \sin\theta$
$\frac{r}{7} = \frac{y}{r}$
$r^2 = 7y$
$x^2 + y^2 = 7y$
$x^2 + y^2 - 7y = 0$
$x^2 + y^2 - 7y + \frac{49}{4} = \frac{49}{4}$
$\boxed{x^2 + (y - \frac{7}{2})^2 = \frac{49}{4}}$

5. $r = -3\sec\theta$
$r = \frac{-3}{\cos\theta}$
$r\cos\theta = -3$
$x = r\cos\theta$
$\boxed{x = -3}$

6. $r = 5\cos\theta$
$\frac{r}{5} = \cos\theta$
$\frac{x}{r} = \cos\theta$
$r = 5(\frac{x}{r})$
$r^2 = 5x$
$x^2 + y^2 = 5x$
$x^2 - 5x + \frac{25}{4} + y^2 = 0 + \frac{25}{4}$
$\boxed{(x - \frac{5}{2})^2 + y^2 = \frac{25}{4}}$

7. $x^2 + y^2 = 10$
$r^2 = 10$
$\boxed{r = \sqrt{10}}$

8. $2x - y^2 = 0$
$2x = y^2$
$2(r\cos\theta) = (r\sin\theta)^2$
$2r\cos\theta = r^2\sin^2\theta$
$2\cos\theta = r\sin^2\theta$
$\frac{2\cos\theta}{\sin^2\theta} = r$
$\boxed{2\cos\theta\csc^2\theta = r}$

9. $x^2 + y^2 = 81$
$r^2 = 81$
$\boxed{r = 9}$

10. $y = -5$
$y = r\sin\theta$
$-5 = r\sin\theta$
$\boxed{r = -5\csc\theta}$

11. $y^2 = 10x$
$(r\sin\theta)^2 = 10r\cos\theta$
$r^2\sin^2\theta = 10r\cos\theta$
$r\sin^2\theta = 10\cos\theta$
~~$r = 10\cos\theta\sec^2\theta$~~
$\boxed{r = 10\cos\theta\csc^2\theta}$

12. $3xy = 7$
$3(r\cos\theta)(r\sin\theta) = 7$
$r^2\, 3\cos\theta\sin\theta = 7$
$\boxed{r^2 = \frac{7}{3}\sec\theta\csc\theta}$